

**STATISTICS:**

a series of **TEXTBOOKS** and **MONOGRAPHS**

# Statistical Design of Experiments with Engineering Applications

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**Chapman & Hall/CRC**  
Taylor & Francis Group

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