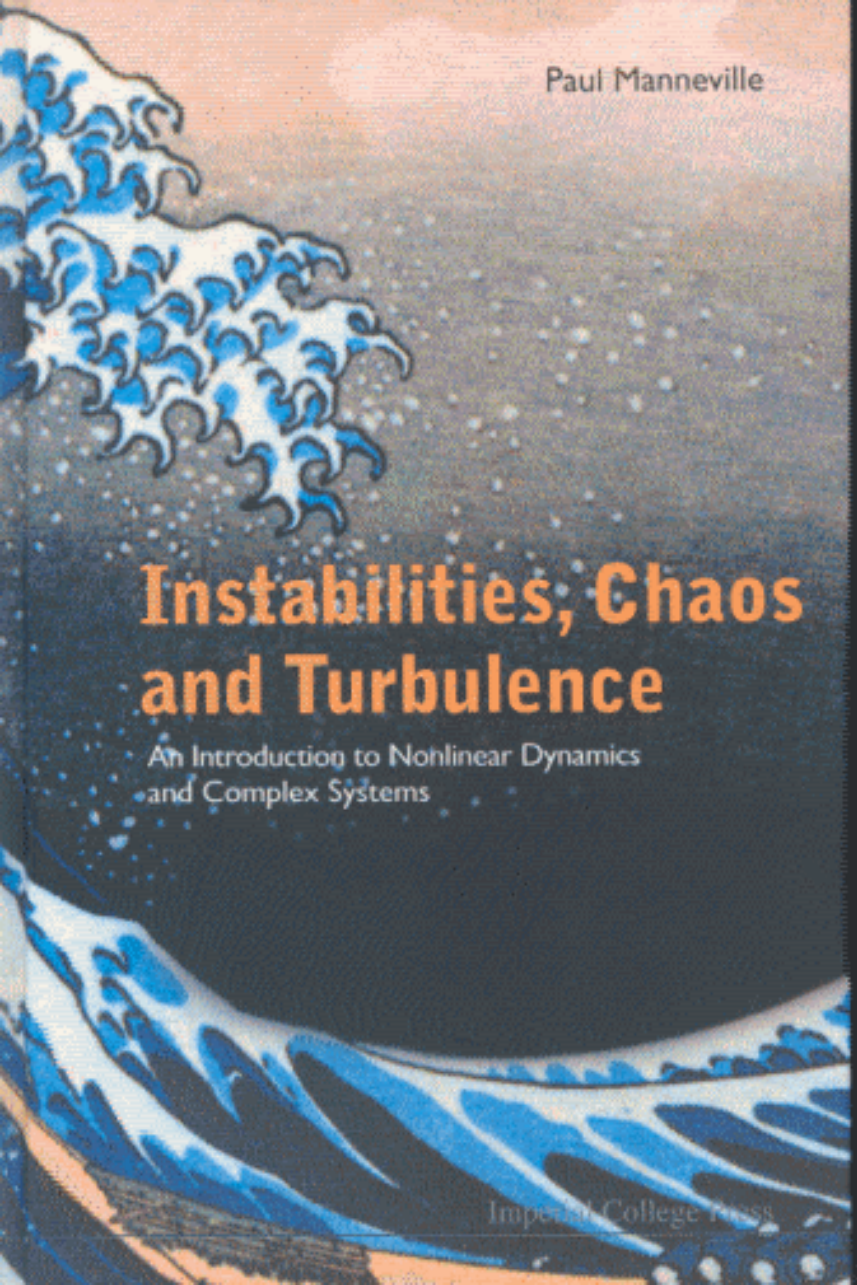




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Instabilities, Chaos and Turbulence

An Introduction to Nonlinear Dynamics
and Complex Systems

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