

**8**<sup>TH</sup>  
EDITION



**ERWIN  
KREYSZIG**

# **ADVANCED ENGINEERING MATHEMATICS**

Abridged International Student Edition

**Authorized For Sale**  
I n A s i a O n l y

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