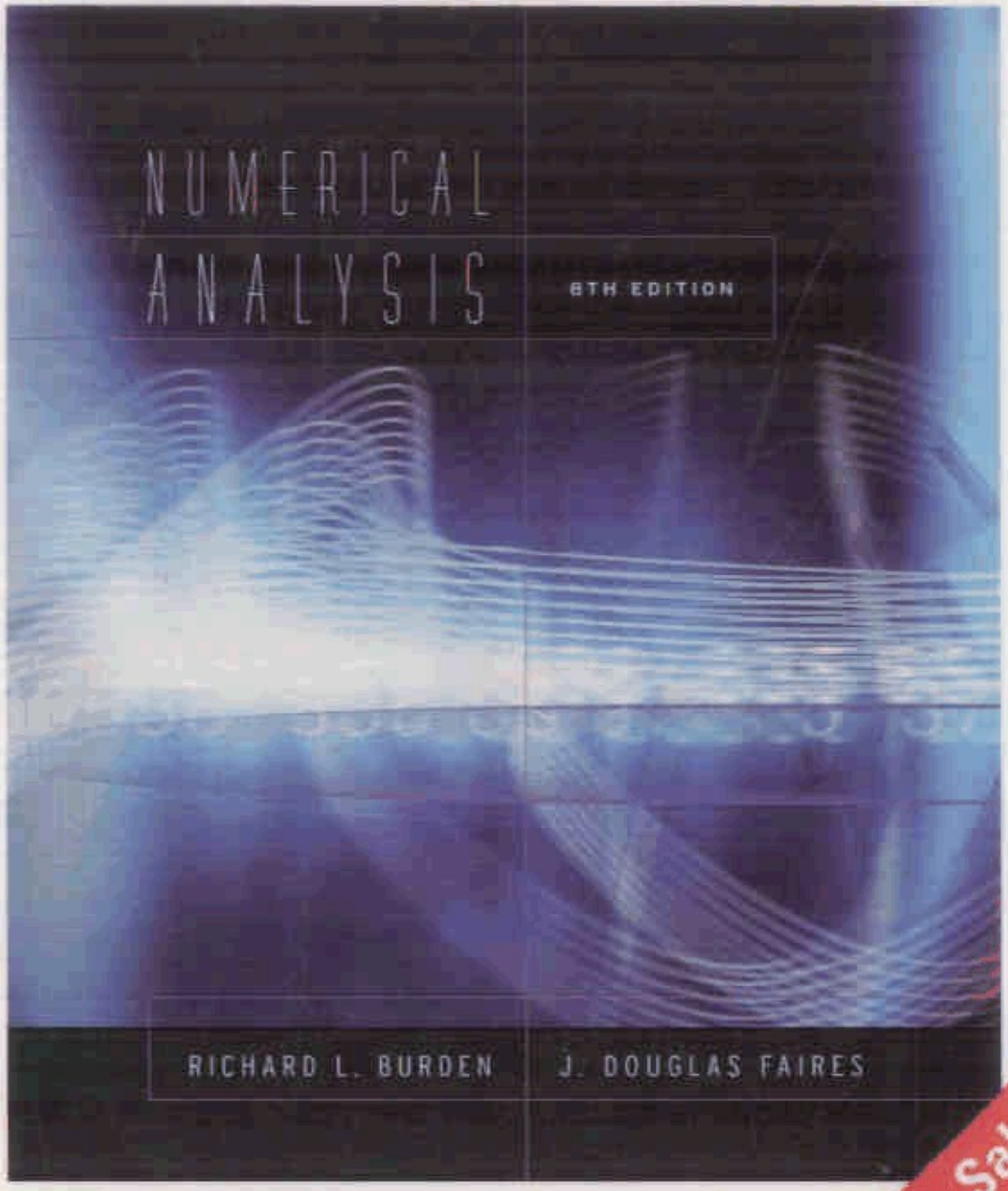
INTERNATIONAL STUDENT EDITION



Not for Sale in the United States

Contents

1 Mathematical Preliminaries and Error Analysis 1

1.1 Review of Calculus 2
1.2 Round-off Errors and Computer Arithmetic 17
1.3 Algorithms and Convergence 30
1.4 Numerical Software 38

2 Solutions of Equations in One Variable 45

2.1 The Bisection Method 46
2.2 Fixed-Point Iteration 53
2.3 Newton's Method 63
2.4 Error Analysis for Iterative Methods 75
2.5 Accelerating Convergence 83
2.6 Zeros of Polynomials and Müller's Method 87
2.7 Survey of Methods and Software 97

3 Interpolation and Polynomial Approximation 101

3.1 Interpolation and the Lagrange Polynomial 104
3.2 Divided Differences 119
3.3 Hermite Interpolation 130
3.4 Cubic Spline Interpolation 137
3.5 Parametric Curves 157
3.6 Survey of Methods and Software 164

4 Numerical Differentiation and Integration 167

4.1 Numerical Differentiation 168
4.2 Richardson's Extrapolation 179
4.3 Elements of Numerical Integration 187
4.4 Composite Numerical Integration 196

4.5 Romberg Integration 207
4.6 Adaptive Quadrature Methods 212
4.7 Gaussian Quadrature 220
4.8 Multiple Integrals 226
4.9 Improper Integrals 241
4.10 Survey of Methods and Software 246

5 Initial-Value Problems for Ordinary Differential Equations 249

5.1 The Elementary Theory of Initial-Value Problems 250
5.2 Euler's Method 256
5.3 Higher-Order Taylor Methods 266
5.4 Runge–Kutta Methods 273
5.5 Error Control and the The Runge–Kutta–Fehlberg Method 283
5.6 Multistep Methods 291
5.7 Variable Step-Size Multistep Methods 302
5.8 Extrapolation Methods 308
5.9 Higher-Order Equations and Systems of Differential Equations 313
5.10 Stability 325
5.11 Stiff Differential Equations 335
5.12 Survey of Methods and Software 342

6 Direct Methods for Solving Linear Systems 345

6.1 Linear Systems of Equations 346
6.2 Pivoting Strategies 360
6.3 Linear Algebra and Matrix Inversion 370
6.4 The Determinant of a Matrix 383
6.5 Matrix Factorization 388
6.6 Special Types of Matrices 398
6.7 Survey of Methods and Software 413

7 Iterative Techniques in Matrix Algebra 417

7.1 Norms of Vectors and Matrices 418
7.2 Eigenvalues and Eigenvectors 429
7.3 Iterative Techniques for Solving Linear Systems 436
7.4 Error Bounds and Iterative Refinement 454
7.5 The Conjugate Gradient Method 464
7.6 Survey of Methods and Software 479

8 Approximation Theory 481

8.1 Discrete Least Squares Approximation 482
8.2 Orthogonal Polynomials and Least Squares Approximation 494
8.3 Chebyshev Polynomials and Economization of Power Series 503
8.4 Rational Function Approximation 512
8.5 Trigonometric Polynomial Approximation 523
8.6 Fast Fourier Transforms 532
8.7 Survey of Methods and Software 544

9 Approximating Eigenvalues 547

9.1 Linear Algebra and Eigenvalues 548
9.2 The Power Method 557
9.3 Householder's Method 574
9.4 The QR Algorithm 582
9.5 Survey of Methods and Software 594

10 Numerical Solutions of Nonlinear Systems of Equations 597

10.1 Fixed Points for Functions of Several Variables 598
10.2 Newton's Method 607
10.3 Quasi-Newton Methods 617
10.4 Steepest Descent Techniques 624
10.5 Homotopy and Continuation Methods 631
10.6 Survey of Methods and Software 639

11 Boundary-Value Problems for Ordinary Differential Equations 641

11.1 The Linear Shooting Method 642
11.2 The Shooting Method for Nonlinear Problems 649
11.3 Finite-Difference Methods for Linear Problems 656
11.4 Finite-Difference Methods for Nonlinear Problems 662
11.5 The Rayleigh–Ritz Method 668
11.6 Survey of Methods and Software 684

12 Numerical Solutions to Partial Differential Equations 687

12.1 Elliptic Partial Differential Equations 690
12.2 Parabolic Partial Differential Equations 699
12.3 Hyperbolic Partial Differential Equations 713
12.4 An Introduction to the Finite-Element Method 721
12.5 Survey of Methods and Software 735

Bibliography 737

Answers to Selected Exercises 747

Index 837