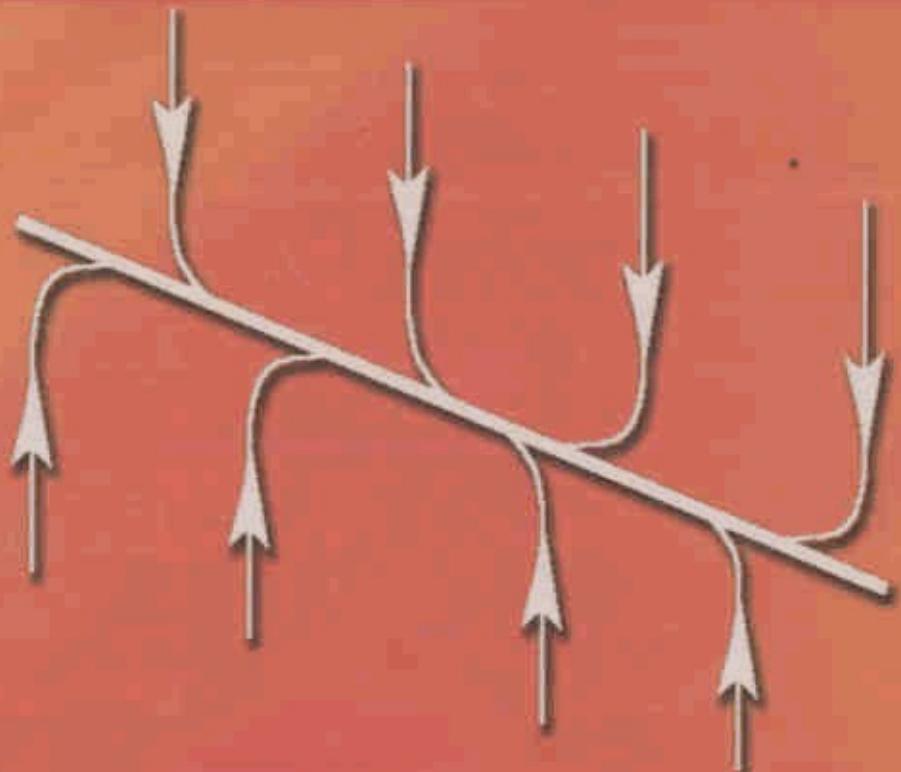




# Design of Nonlinear Control Systems with the Highest Derivative in Feedback

Valery D. Yurkevich



World Scientific

# Contents

<i>Preface</i>	vii
1. Regularly and singularly perturbed systems	1
1.1 Regularly perturbed systems . . . . .	1
1.1.1 Nonlinear nominal system . . . . .	1
1.1.2 Linear nominal system . . . . .	3
1.1.3 Vanishing perturbation . . . . .	5
1.1.4 Nonvanishing perturbation . . . . .	6
1.2 Singularly perturbed systems . . . . .	7
1.2.1 Singular perturbation . . . . .	7
1.2.2 Two-time-scale motions . . . . .	8
1.2.3 Boundary-layer system . . . . .	10
1.2.4 Stability analysis . . . . .	10
1.2.5 Fast and slow-motion subsystems . . . . .	14
1.2.6 Degree of time-scale separation . . . . .	15
1.3 Discrete-time singularly perturbed systems . . . . .	18
1.3.1 Fast and slow-motion subsystems . . . . .	18
1.3.2 Degree of time-scale separation . . . . .	19
1.4 Notes . . . . .	21
1.5 Exercises . . . . .	21
2. Design goal and reference model	23
2.1 Design goal . . . . .	23
2.2 Basic step response parameters . . . . .	24
2.3 Reference model . . . . .	25
2.4 Notes . . . . .	30

2.5 Exercises . . . . .	31
3. Methods of control system design under uncertainty	33
3.1 Desired vector field in the state space of plant model . . . . .	33
3.2 Solution of nonlinear inverse dynamics . . . . .	36
3.3 The highest derivative and high gain in feedback loop . . . . .	37
3.4 Differentiating filter and high-gain observer . . . . .	40
3.5 Influence of noise in control system with the highest derivative	43
3.6 Desired manifold in the state space of plant model . . . . .	45
3.7 State vector and high gain in feedback loop . . . . .	46
3.8 Control systems with sliding motions . . . . .	49
3.9 Example . . . . .	52
3.10 Notes . . . . .	54
3.11 Exercises . . . . .	55
4. Design of SISO continuous-time control systems	57
4.1 Controller design for plant model of the 1st order . . . . .	57
4.1.1 Control problem . . . . .	57
4.1.2 Insensitivity condition . . . . .	58
4.1.3 Control law with the 1st derivative in feedback loop	59
4.1.4 Closed-loop system properties . . . . .	61
4.2 Controller design for an $n$ th-order plant model . . . . .	64
4.2.1 Control problem . . . . .	64
4.2.2 Insensitivity condition . . . . .	65
4.2.3 Control law with the $n$ th derivative in the feedback loop	66
4.2.4 Fast-motion subsystem . . . . .	68
4.2.5 Slow-motion subsystem . . . . .	72
4.2.6 Influence of small parameter . . . . .	73
4.2.7 Geometric interpretation of control problem solution	74
4.3 Example . . . . .	75
4.4 Notes . . . . .	76
4.5 Exercises . . . . .	77
5. Advanced design of SISO continuous-time control systems	79
5.1 Control accuracy . . . . .	79
5.1.1 Steady state of fast-motion subsystem . . . . .	79
5.1.2 Steady state of slow-motion subsystem . . . . .	80

5.1.3	Velocity error due to external disturbance . . . . .	82
5.1.4	Velocity error due to reference input . . . . .	83
5.1.5	Control law in the form of forward compensator . .	84
5.2	Root placement of FMS characteristic polynomial . . . .	85
5.2.1	Degree of time-scale separation . . . . .	85
5.2.2	Selection of controller parameters . . . . .	86
5.2.3	Root placement based on normalized polynomials .	87
5.3	Bode amplitude diagram assignment of closed-loop FMS .	88
5.3.1	Block diagram of closed-loop system . . . . .	88
5.3.2	Bode amplitude diagram of closed-loop FMS . . . .	89
5.3.3	Desired Bode amplitude diagram of closed-loop FMS	91
5.3.4	Selection of controller parameters . . . . .	92
5.4	Influence of high-frequency sensor noise . . . . .	93
5.4.1	Closed-loop system in presence of sensor noise . .	93
5.4.2	Controller with infinite bandwidth . . . . .	94
5.4.3	Controller with finite bandwidth . . . . .	96
5.5	Influence of varying parameters . . . . .	100
5.5.1	Influence of varying parameters on FMS and SMS .	100
5.5.2	Michailov hodograph for FMS . . . . .	100
5.5.3	Variation of FMS bandwidth . . . . .	102
5.5.4	Degree of control law differential equation . . . . .	103
5.5.5	Root placement of FMS characteristic polynomial .	104
5.6	Bode amplitude diagram assignment of open-loop FMS .	105
5.7	Relation with PD, PI, and PID controllers . . . . .	107
5.8	Example . . . . .	109
5.9	Notes . . . . .	111
5.10	Exercises . . . . .	112
6.	Influence of unmodeled dynamics	115
6.1	Pure time delay . . . . .	116
6.1.1	Plant model with pure time delay in control . . . .	116
6.1.2	Closed-loop system with delay in feedback loop .	117
6.1.3	Fast motions in presence of delay . . . . .	118
6.1.4	Stability of FMS with delay . . . . .	119
6.1.5	Phase margin of FMS with delay . . . . .	121
6.1.6	Control with compensation of delay . . . . .	122
6.1.7	Velocity error with respect to external disturbance .	124
6.1.8	Example . . . . .	124
6.2	Regular perturbances . . . . .	126

6.2.1	Regularly perturbed plant model . . . . .	126
6.2.2	Fast motions in presence of regular perturbances . .	127
6.2.3	Selection of controller parameters . . . . .	128
6.2.4	Control with compensation of regular perturbances	129
6.2.5	Example . . . . .	130
6.3	Singular perturbances . . . . .	132
6.3.1	Singularly perturbed plant model . . . . .	132
6.3.2	Fast motions in presence of singular perturbances .	133
6.3.3	Selection of controller parameters . . . . .	134
6.4	Nonsmooth nonlinearity in control loop . . . . .	136
6.4.1	System preceded by nonsmooth nonlinearity . . . .	136
6.4.2	Describing function analysis of limit cycle in FMS .	138
6.4.3	Effect of chattering on control accuracy . . . . .	141
6.4.4	Example . . . . .	143
6.5	Notes . . . . .	145
6.6	Exercises . . . . .	146
7.	Realizability of desired output behavior	149
7.1	Control problem statement for MIMO control system . . .	149
7.1.1	MIMO plant model . . . . .	149
7.1.2	Control problem . . . . .	150
7.2	Invertibility of dynamical systems . . . . .	151
7.2.1	Role of invertibility of dynamical systems . . . . .	151
7.2.2	Definition of invertibility of dynamic control system	152
7.2.3	Invertibility condition for nonlinear systems . . . .	154
7.3	Insensitivity condition for MIMO control system . . . .	157
7.3.1	Desired dynamics equations . . . . .	157
7.3.2	Insensitivity condition . . . . .	158
7.4	Internal stability . . . . .	159
7.4.1	Boundedness of <i>NID</i> -control function . . . . .	159
7.4.2	Concept of internal stability . . . . .	160
7.4.3	Normal form of the plant model . . . . .	161
7.4.4	Internal stability of linear systems . . . . .	164
7.4.5	Internal stability of nonlinear systems . . . . .	167
7.4.6	Degenerated motions and zero-dynamics . . . . .	168
7.4.7	Example . . . . .	170
7.5	Output regulation of SISO systems . . . . .	171
7.5.1	Realizability of desired output behavior . . . . .	171
7.5.2	Closed-loop system analysis . . . . .	174

7.5.3	Example . . . . .	175
7.6	Switching regulator for boost DC-to-DC converter . . . . .	176
7.6.1	Boost DC-to-DC converter circuit model . . . . .	176
7.6.2	Model with continuous control variable . . . . .	177
7.6.3	Switching regulator . . . . .	180
7.6.4	External disturbance attenuation . . . . .	183
7.7	Notes . . . . .	185
7.8	Exercises . . . . .	186
8.	Design of MIMO continuous-time control systems	189
8.1	MIMO system without internal dynamics . . . . .	189
8.1.1	MIMO system with identical relative degrees . . . . .	189
8.1.2	MIMO system with different relative degrees . . . . .	191
8.2	MIMO control system design (identical relative degrees) .	192
8.2.1	Insensitivity condition . . . . .	192
8.2.2	Control system with the relative highest derivatives in feedback . . . . .	194
8.2.3	Fast-motion subsystem . . . . .	195
8.2.4	Slow-motion subsystem . . . . .	197
8.2.5	Control system design with zero steady-state error .	198
8.2.6	Example . . . . .	200
8.3	MIMO control system design (different relative degrees) .	202
8.3.1	Insensitivity condition and control law structure .	202
8.3.2	Closed-loop system analysis . . . . .	203
8.3.3	Control accuracy . . . . .	205
8.4	MIMO control system in presence of internal dynamics .	207
8.4.1	Fast-motion subsystem . . . . .	209
8.4.2	Slow-motion subsystem . . . . .	210
8.4.3	Example . . . . .	211
8.5	Decentralized output feedback controller . . . . .	212
8.6	Notes . . . . .	214
8.7	Exercises . . . . .	215
9.	Stabilization of internal dynamics	217
9.1	Zero placement by redundant control . . . . .	217
9.2	Internal dynamics stabilization (particular case) . . . . .	221
9.3	Internal dynamics stabilization (generalized case) . . . . .	222
9.4	Stabilization of degenerated mode and zero dynamics .	225

9.5	Methods of internal dynamics stabilization . . . . .	225
9.6	Example . . . . .	228
9.7	Notes . . . . .	231
9.8	Exercises . . . . .	232
10.	Digital controller design based on pseudo-continuous approach	233
10.1	Continuous system preceded by zero-order hold . . . . .	233
10.1.1	Control problem . . . . .	233
10.1.2	Pseudo-continuous-time model with pure delay . . . . .	234
10.2	Digital controller design . . . . .	235
10.2.1	Insensitivity condition . . . . .	235
10.2.2	Pseudo-continuous closed-loop system . . . . .	236
10.2.3	Influence of sampling period . . . . .	237
10.2.4	Digital realization of continuous controller . . . . .	239
10.2.5	Example . . . . .	242
10.3	Digital controller design with compensation of delay . . . . .	242
10.3.1	Control law structure . . . . .	242
10.3.2	Closed-loop system analysis . . . . .	244
10.3.3	Digital realization of continuous controller . . . . .	245
10.3.4	Example . . . . .	246
10.4	Notes . . . . .	248
10.5	Exercises . . . . .	250
11.	Design of discrete-time control systems	253
11.1	SISO two-time-scale discrete-time control systems . . . . .	253
11.1.1	Discrete-time systems . . . . .	253
11.1.2	Control problem and insensitivity condition . . . . .	254
11.1.3	Discrete-time control law . . . . .	256
11.1.4	Two-time-scale motion analysis . . . . .	257
11.1.5	Robustness of closed-loop system properties . . . . .	260
11.1.6	Control accuracy . . . . .	262
11.1.7	Example . . . . .	265
11.2	SISO discrete-time control systems with small parameter . . . . .	266
11.2.1	System with small parameter . . . . .	266
11.2.2	Two-time-scale motion analysis . . . . .	268
11.2.3	Interrelationship with fixed point theorem . . . . .	271
11.2.4	Root placement of FMS characteristic polynomial . . . . .	273
11.2.5	FMS design based on frequency-domain methods . . . . .	274

11.3 MIMO two-time-scale discrete-time control systems . . . . .	278
11.3.1 MIMO discrete-time systems . . . . .	278
11.3.2 Control law . . . . .	278
11.3.3 Two-time-scale motion analysis . . . . .	281
11.3.4 Example . . . . .	283
11.4 Notes . . . . .	284
11.5 Exercises . . . . .	285
 12. Design of sampled-data control systems	287
12.1 SISO sampled-data control systems . . . . .	287
12.1.1 Reduced order pulse transfer function . . . . .	287
12.1.2 Input-output approximate model of linear system .	290
12.1.3 Control law . . . . .	291
12.1.4 Closed-loop system analysis . . . . .	293
12.1.5 Selection of controller parameters . . . . .	296
12.1.6 Nonlinear sampled-data systems . . . . .	297
12.1.7 Example . . . . .	300
12.2 MIMO sampled-data control systems . . . . .	300
12.2.1 Control problem . . . . .	300
12.2.2 MIMO continuous-time system preceded by ZOH .	302
12.2.3 Control law . . . . .	303
12.2.4 Fast-motion subsystem . . . . .	305
12.2.5 Selection of controller parameters . . . . .	306
12.2.6 Slow-motion subsystem . . . . .	307
12.2.7 Example . . . . .	308
12.3 Notes . . . . .	309
12.4 Exercises . . . . .	311
 13. Control of distributed parameter systems	313
13.1 One-dimensional heat system with distributed control . . .	313
13.2 Heat system with finite-dimensional control . . . . .	317
13.3 Degenerated motions . . . . .	321
13.4 Estimation of modes . . . . .	322
13.5 Notes . . . . .	323
13.6 Exercises . . . . .	323
 Appendix A Proofs	325
A.1 Proof of expression (8.29) . . . . .	325

A.2 Proof of expression (8.42) . . . . .	325
A.3 Proof of expression (8.65) . . . . .	326
A.4 Proof of expression (11.37) . . . . .	327
A.5 Proof of expressions (11.40)–(11.41) . . . . .	328
A.6 Proof of expression (11.47) . . . . .	328
A.7 Proof of expression (11.51) . . . . .	330
A.8 Proof of expression (12.56) . . . . .	332
A.9 Proof of expression (12.57) . . . . .	333
Appendix B Notation system	335
<i>Bibliography</i>	337
<i>Index</i>	349