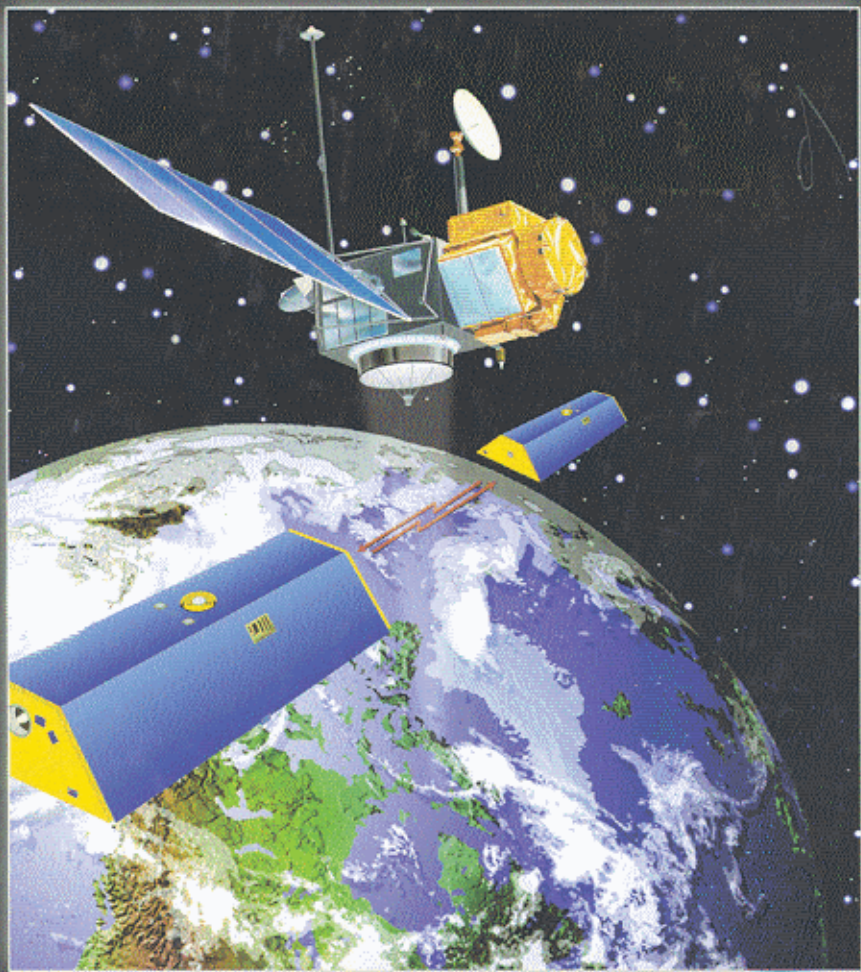
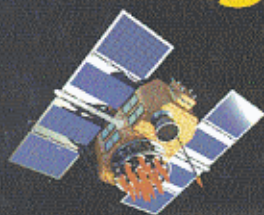


Statistical Orbit Determination



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Contents

Preface	xi
1 Orbit Determination Concepts	1
1.1 Introduction	1
1.2 Uniform Gravity Field Model	3
1.2.1 Formulation of the Problem	4
1.2.2 The Equation of the Orbit	5
1.2.3 The Role of the Observation	5
1.2.4 Linearization Procedure	9
1.2.5 State Transition Matrix	11
1.3 Background and Overview	13
1.4 Summary	14
1.5 References	15
1.6 Exercises	16
2 The Orbit Problem	17
2.1 Historical Background	17
2.2 Problem of Two Bodies: General Properties	19
2.2.1 Motion in the Plane	21
2.2.2 Motion in Space	28
2.2.3 Basic Coordinate Systems	29
2.2.4 Orbit Elements and Position/Velocity	32
2.2.5 Position/Velocity Prediction	35
2.2.6 State Transition Matrix and Error Propagation	40
2.3 Perturbed Motion	44
2.3.1 Classical Example: Lunar Problem	45
2.3.2 Variation of Parameters	48
2.3.3 Gravitational Perturbations: Mass Distribution	50
2.3.4 Gravitational Perturbations: Oblateness and Other Effects	56
2.3.5 Gravitational Perturbations: Third-Body Effects	61

2.3.6	Gravitational Perturbations: Temporal Changes in Gravity	64
2.3.7	Gravitational Perturbations: General Relativity	65
2.3.8	Nongravitational Perturbations: Atmospheric Resistance	66
2.3.9	Nongravitational Perturbations: Radiation Pressure	68
2.3.10	Nongravitational Perturbations: Other	68
2.3.11	Perturbed Equations of Motion: Summary	69
2.4	Coordinate Systems and Time: Introduction	71
2.4.1	Precession and Nutation	71
2.4.2	Earth Rotation and Time	74
2.4.3	Earth-Fixed and Topocentric Systems	77
2.4.4	Transformation between ECF and ECI	82
2.5	Orbit Accuracy	82
2.6	References	84
2.7	Exercises	87
3	Observations	93
3.1	Introduction	93
3.2	Observations	94
3.2.1	Ideal Range	94
3.2.2	Ideal Range-Rate	95
3.2.3	Ideal Azimuth and Elevation Angles	95
3.2.4	Examples: Ideal Observations	95
3.3	Conceptual Measurement Systems	99
3.3.1	Range	99
3.3.2	Range-Rate	107
3.4	Realization of Measurements	110
3.4.1	Considerations	110
3.4.2	Atmospheric Effects	110
3.4.3	General Relativity	115
3.5	Measurement Systems	116
3.5.1	One-Way Range	116
3.5.2	Two-Way Range	125
3.5.3	Doppler Systems	132
3.5.4	Examples	134
3.6	Differenced Measurements	139
3.6.1	Differenced GPS Measurements	140
3.6.2	Differenced Altimeter Data	145
3.7	Satellite Positions	147
3.8	Angles	148
3.9	References	149
3.10	Exercises	151

4	Fundamentals of Orbit Determination	159
4.1	Introduction	159
4.2	Linearization of the Orbit Determination Process	160
4.2.1	The State Transition Matrix	164
4.2.2	Solution for the State Transition Matrix	167
4.2.3	Relating the Observations to an Epoch State	172
4.3	The Least Squares Solution	173
4.3.1	The Minimum Norm Solution	174
4.3.2	Shortcomings of the Least Squares Solution	176
4.3.3	Weighted Least Squares Solution	176
4.3.4	An Alternate Least Squares Approach	178
4.4	The Minimum Variance Estimate	183
4.4.1	Propagation of the Estimate and Covariance Matrix	188
4.4.2	Minimum Variance Estimate with <i>A Priori</i> Information	188
4.5	Maximum Likelihood and Bayesian Estimation	190
4.6	Computational Algorithm for the Batch Processor	194
4.7	The Sequential Estimation Algorithm	199
4.7.1	The Sequential Computational Algorithm	201
4.7.2	The Extended Sequential Estimation Algorithm	209
4.7.3	The Extended Sequential Computational Algorithm	210
4.7.4	The Prediction Residual	210
4.8	Example Problems	211
4.8.1	Linear System	211
4.8.2	Spring-Mass Problem	216
4.9	State Noise Compensation Algorithm	220
4.9.1	The Gauss-Markov Process	230
4.10	Information Filter	233
4.11	Batch and Sequential Estimation	237
4.12	Observability	237
4.13	Error Sources	241
4.14	Orbit Accuracy	244
4.15	Smoothing	246
4.15.1	Computational Algorithm for Smoother	250
4.16	The Probability Ellipsoid	251
4.16.1	Transformation of the Covariance Matrix between Coordinate Systems	257
4.17	Combining Estimates	258
4.18	References	261
4.19	Exercises	264

5	Square Root Solution Methods	285
5.1	Introduction	285
5.2	Cholesky Decomposition	286
5.2.1	The Cholesky Algorithm	287
5.2.2	The Square Root Free Cholesky Algorithm	288
5.3	Least Squares Solution via Orthogonal Transformation	290
5.4	Givens Transformations	291
5.4.1	<i>A Priori</i> Information and Initialization	294
5.4.2	Givens Computational Algorithm	298
5.4.3	Square Root Free Givens Transformation	301
5.4.4	Square Root Free Givens Computational Algorithm	305
5.4.5	A Simplified Square Root Free Givens Transformation	307
5.4.6	Implementation Considerations	309
5.5	The Householder Transformation	310
5.5.1	Application to the Solution of the Least Squares Problem	315
5.5.2	Householder Computational Algorithm	316
5.6	Numerical Examples	318
5.6.1	Cholesky Decomposition	318
5.6.2	Givens Transformation	320
5.6.3	Householder Transformation	322
5.6.4	A More Illustrative Example	324
5.6.5	Cholesky Decomposition	325
5.6.6	Square Root Free Givens Transformation	326
5.6.7	The Householder Transformation	329
5.7	Square Root Filter Algorithms	330
5.7.1	The Square Root Measurement Update Algorithms	332
5.7.2	Square Root Free Measurement Update Algorithms	340
5.8	Time Update of the Estimation Error Covariance Matrix	343
5.9	Continuous State Error Covariance Propagation	346
5.9.1	Triangular Square Root Algorithm	349
5.10	The Square Root Information Filter	351
5.10.1	The Square Root Information Filter with Time-Dependent Effects	353
5.10.2	The Dynamic Case with Process Noise	358
5.10.3	SRIF Computational Algorithm	363
5.10.4	Smoothing with the SRIF	364
5.11	Process Noise Parameter Filtering/Smoothing Using a SRIF	369
5.11.1	Exponentially Correlated Process Noise SRIF	371
5.11.2	Smoothing with a SRIF	377
5.12	References	380
5.13	Exercises	381

6	Consider Covariance Analysis	387
6.1	Introduction	387
6.2	Bias in Linear Estimation Problems	388
6.3	Formulation of the Consider Covariance Matrix	389
6.4	The Sensitivity and Perturbation Matrices	397
6.4.1	Example Application of a Sensitivity and Perturbation Matrix	398
6.5	Inclusion of Time-Dependent Effects	400
6.6	Propagation of the Error Covariance	405
6.7	Sequential Consider Covariance Analysis	407
6.8	Example: Freely Falling Point Mass	410
6.8.1	Propagation with Time	415
6.8.2	The Sequential Consider Algorithm	416
6.8.3	Perturbation in the State Estimate	419
6.9	Example: Spring-Mass Problem	420
6.10	Errors in the Observation Noise and <i>A Priori</i> State Covariances	425
6.11	Errors in Process Noise, Observation Noise, and State Covariance	427
6.12	Covariance Analysis and Orthogonal Transformations	430
6.13	References	434
6.14	Exercises	435
A	Probability and Statistics	439
A.1	Introduction	439
A.2	Axioms of Probability	440
A.3	Conditional Probability	443
A.4	Probability Density and Distribution Functions	443
A.5	Expected Values	445
A.6	Examples and Discussion of Expectation	446
A.7	Moment Generating Functions	448
A.8	Some Important Continuous Distributions	449
A.8.1	Uniform or Rectangular Distribution	449
A.8.2	The Gaussian or Normal Distribution	450
A.9	Two Random Variables	452
A.10	Marginal Distributions	453
A.11	Independence of Random Variables	454
A.12	Conditional Probability	454
A.13	Expected Values of Bivariate Functions	455
A.14	The Variance-Covariance Matrix	456
A.15	Properties of the Correlation Coefficient	458
A.16	Properties of Covariance and Correlation	460
A.17	Bivariate Normal Distribution	460
A.18	Marginal Distributions	461

A.19	The Multivariate Normal Distribution	462
A.19.1	The Conditional Distribution for Multivariate Normal Variables	464
A.20	The Central Limit Theorem	465
A.21	Bayes Theorem	465
A.22	Stochastic Processes	467
A.22.1	Definitions for Stochastic Processes	469
A.23	References	471
B	Review of Matrix Concepts	473
B.1	Introduction	473
B.2	Rank	475
B.3	Quadratic Forms	475
B.4	Determinants	476
B.5	Matrix Trace	477
B.6	Eigenvalues and Eigenvectors	478
B.7	The Derivatives of Matrices and Vectors	478
B.8	Maxima and Minima	480
B.9	Useful Matrix Inversion Theorems	481
B.10	Reference	483
C	Equations of Motion	485
C.1	Lagrange Planetary Equations	485
C.2	Gaussian Form	485
C.3	References	486
D	Constants	487
D.1	Physical Constants	487
D.2	Earth Constants	487
D.3	Lunar, Solar, and Planetary Masses	488
D.4	References	490
E	Analytical Theory for Near-Circular Orbits	493
E.1	Description	493
E.2	Example	497
E.3	References	497
F	Example of State Noise and Dynamic Model Compensation	499
F.1	Introduction	499
F.2	State Noise Compensation	501
F.3	Dynamic Model Compensation	505
F.4	Reference	509

G	Solution of the Linearized Equations of Motion	511
G.1	Introduction	511
G.2	The State Transition Matrix	513
H	Transformation between ECI and ECF Coordinates	517
H.1	Introduction	517
H.2	Matrix P	518
H.3	Matrix N	519
H.4	Matrix S'	519
H.5	Matrix W	521
H.6	References	521
	Bibliography Abbreviations	523
	Bibliography	525
	Author Index	537
	Index	541