Remote State Preparation and Operation for Photons

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Abstract
Remote state preparation and remote operation are entanglement-assisted protocols in quantum information process, here we present a practical and general scheme of remote preparation for pure and mixed state, in which an auxiliary qubit and controlled-NOT gate are used. We give an experimental scheme of the quantum remote operation on single photons, where the unitary operation is restricted to the sets $U_{com}$ or $U_{anti}$.

We discuss the remote state preparation (RSP) in two important types of decoherent channel (depolaring and dephasing). In our experiment, we realize RSP in the dephasing channel by using spontaneous parametric down conversion (SPDC), linear optical elements and single photon detector.

1 Introduction

Entanglement is not only one of the most fantastic properties of quantum systems, but a powerful resource in quantum information processing[1]. An example of entanglement-assisted process is provided by quantum-state teleportation[2], where an arbitrary qubit state can be transferred with perfect fidelity among distant parties with two classical communication bits (2 cbits) and one maximally entangled state. Recently, a related problem, remote state preparation (RSP)[3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and quantum remote operation[13, 14, 15, 16], attract the interest of many scientists.

The former[3, 4, 5] allows a state, randomly chosen by the sender (Alice) from a pre-agreed set, to be reproduced perfectly on the side of the receiver (Bob), who know nothing about the state except the set. The total cost is one maximal entanglement (1 ebit) shared and one bit of classical communication (1 cibit), while one ebit and two cbits for standard state teleportation. Recently, the RSP protocol is generalized from pure states to mixed ones. The essence is to replace von Neumann measurement by positive operator-valued measure (POVM)[7].

In quantum remote operation[13, 14, 15], Alice wants to implement an operation $U$ on an remote particle in arbitrary state $|\psi\rangle$ which is on Bob’s side by
local operation, classical communication and shared entanglement. The trivial implementation of a remote arbitrary unitary operation can be realized by bidirectional state teleportation. First the state is teleported from Bob to Alice. Then Alice applies the operation on the state she received to get a new state. And finally the new one is sent from Alice to Bob via another state teleportation. The total resource for this trivial protocol is two ebits shared and two cbits in each direction. If \( U \) is restricted in the unitary operation set \( U \equiv U_{\text{anti}} \cup U_{\text{anti}} \), where \( U_{\text{com}} \) (\( U_{\text{anti}} \)) which (anti-)commutes with \( \sigma_z \), the resource can be reduced to two ebits shared, two cbits from B to A and one cbit from A to B in the optimal nontrivial scheme. Further, if the set is either \( U_{\text{com}} \) or \( U_{\text{anti}} \), the scheme can be simplified by using only one ebit shared and one cbit in each direction. Further, the RSP protocol can be considered as a special case of remote operation on a fixed initial state.

In this paper, we present a scheme for remote preparation of mixed states (including pure ones) and realize it for the polarization states of single photons. Two types of decoherence, depolarizing and dephasing, are discussed by using the state fidelity in theory as well as in our experiment. And we present a scheme to implement remote operation.

This paper is organized as following. A RSP scheme of mixed states is presented in section II. We give an experimental scheme of quantum remote operation in section III, where the operations we discuss here are restricted in the unitary operation set \( U_{\text{com}} \). The effect of decoherence is discussed with state fidelity in section IV. In section V, we carry out an experiment to remotely prepare pure and mixed states with dephasing noisy entanglement. And a conclusion will be given in section VI.

## 2 Remote Preparation of Mixed States

In this section, we discuss remote preparation of mixed state using an maximally entangled state shared by Alice and Bob. An arbitrary state of single qubit can be represented as a vector \( \vec{r} \) on or inside the Bloch sphere (see Fig. 1)

\[
\rho(\vec{r}) = \frac{1}{2} (I + \vec{r} \cdot \vec{\sigma})
\]

where \( \vec{r} = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta) \) and \( 0 \leq r \leq 1 \), \( 0 \leq \theta \leq \pi \), \( 0 \leq \phi \leq 2\pi \). The state is a pure one \( |\psi\rangle = \cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle \) for \( r = 1 \) (on the sphere), or a mixed one for \( r < 1 \) (inside the sphere). Specially, when \( r = 0 \) (zero vector), it is a maximally-mixed one \( \frac{1}{4} (|0\rangle \langle 0| + |1\rangle \langle 1|) \) (zero vector).

In RSP protocol for pure states, a state subset \( \chi \) is known to Alice and Bob. Alice and Bob share a maximally entangled state

\[
|\Psi^-(\text{AB}) = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B).
\]

Particle A (B) is possessed by Alice (Bob) and Alice wants to help Bob prepare a state \( |\psi\rangle \in \chi \) in the distance. Here \( |\psi\rangle \) is selected from \( \chi \) randomly by Alice.
Figure 1: The Bloch sphere. The points on and in it represent the states of a single qubit and vector $\vec{r}$ represent the position of the points.

and is unknown to Bob. For example, $\chi$ can be the equatorial or polar great circle on the Bloch sphere and $|\psi\rangle$ is randomly selected from it.

As we know, $|\Psi^-\rangle_{AB}$ can be expanded in the basis $\{|\psi\rangle, |\psi\perp\rangle\}$,

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}} (|\psi\rangle_A |\psi\perp\rangle_B - |\psi\perp\rangle_A |\psi\rangle_B)$$  \hspace{1em} (3)

where $|\psi\perp\rangle = e^{i\phi} \sin \frac{\theta}{2} |0\rangle - \cos \frac{\theta}{2} |1\rangle$. Alice would perform a von Neumann measurement $\{|\psi\rangle, |\psi\perp\rangle\}$ on her particle $A$ and send the result 0/1 ($|\psi\rangle / |\psi\perp\rangle$) to Bob. Or she can perform a unitary rotation $U(\theta, \phi)^\dagger$ on $A$,

$$U_A(\theta, \phi)^\dagger |\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}} (|0\rangle_A |\psi\perp\rangle_B - |1\rangle_B |\psi\rangle_B)$$  \hspace{1em} (4)

where

$$U(\theta, \phi) = \begin{pmatrix} \cos \theta/2 & -e^{-i\phi} \sin \theta/2 \\ e^{i\phi} \sin \theta/2 & \cos \theta/2 \end{pmatrix}.$$  \hspace{1em} (5)

then carry out a von Neumann measurement $\{|0\rangle, |1\rangle\}$, and send the result 0/1 ($|0\rangle / |1\rangle$) to Bob (see Fig. 2). the result is 1, Bob will find his particle $B$ in $|\psi\rangle$, which is the state Alice wants to prepare. For the result 0, $B$ will be in $|\psi\perp\rangle$ and an operation is need for Bob to flip $|\psi\perp\rangle$ into $|\psi\rangle$. Generally, such operation is unavailable because the universal $NOT$ gate is forbidden for Bob has no knowledge of the state. However, it is possible for some special $\chi$, such as $\sigma_Z$ for the polar greatest circle and $i\sigma_Y$ for the equatorial circle. In the rest of this paper, we only discuss the situation that Alice obtain the result 1.
Figure 2: Schematic protocols for remote preparation of pure state. The EPR state pre-shared by Alice and Bob.

Berry and Sanders[7] have generalized the protocol to mixed state preparation by using POVM instead of von Neumann measurement. We can decompose the mixed state to be prepared (see eq.(1) ) into

$$\rho(\overrightarrow{r}) = \frac{1+r}{2} |\psi\rangle \langle \psi| + \frac{1-r}{2} |\psi_{\perp}\rangle \langle \psi_{\perp}|$$  \hspace{1cm} (6)

To prepare $\rho(\overrightarrow{r})$ remotely, Alice would carry out a two-element POVM $\{\Pi^0, \Pi^1\}$ instead of $\{|0\rangle, |1\rangle\}$,

$$\Pi^1 = \frac{1-r}{2} |0\rangle \langle 0| + \frac{1+r}{2} |1\rangle \langle 1|, \Pi^0 = I - \Pi^1.$$  \hspace{1cm} (7)

If Alice gets the result 1, $B$ will be in $\rho(\overrightarrow{r})$. The essence is the realization of POVM. Here, the POVM can be performed by an auxiliary qubit and $CNOT$ operation. The whole process of RSP (including pure and mixed states) can be divided into the following five steps (see Fig. 3). 1) a unitary rotation $U(\theta, \phi)^\dagger$

Figure 3: Schematic protocols for remote preparation of mixed state. The EPR state pre-shared by Alice and Bob.

on $A$; 2) a $CNOT$ operation $U_{CNOT}$ where $A$ is the controller and the auxiliary qubit $a$ in initial state $|0\rangle$ is the target; 3) another unitary rotation $U_A(r)$ on $A$;
4) a von Neumann measurement on $A$; and 5) sending the result to Bob. And it can be represented as
$$|\Psi^+\rangle_{AB}|0\rangle_a$$
$$U_A = \frac{1}{\sqrt{2}}\begin{pmatrix} |0\rangle & |1\rangle \\
|1\rangle & -|0\rangle 
\end{pmatrix}.$$}

(ii) Alice implements the required operation on qubit $A$. 
$$U_{\text{com}}(\varphi) \otimes I_B |\Psi\rangle_{AB} = \alpha e^{i\varphi} |0\rangle |0\rangle + \beta e^{-i\varphi} |1\rangle |1\rangle$$

It's easy to see that the protocol is valid for pure state by setting $r = 1$.

3 Remote Preparation of A Restricted Unitary Operation

As we known, we can considered the preparation of a state as a special case of implementing an unitary operation. Now, Let's discuss the problem of remote operation.

Let's consider that Alice has an operation $U_{\text{com}}$ to implement and that Bob has an arbitrary state $|\psi\rangle_C$ to be operated, where $U_{\text{com}}(\varphi) = e^{i\varphi \sigma_z}$ and $|\psi\rangle_C = \alpha |0\rangle_C + \beta |1\rangle_C$ ($\{0\}, \{1\}$ are the basis of $\sigma_z$). Given the shared entanglement $|\Phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B$). In [14], a simplified protocol is present remotely implement an arbitrary operation, $U_{\text{anti}}$, commuting with $\sigma_z$ by one ebit shared and one cbit classical communication each direction, finally, one of the Bob's qubits, initially entangled with Alice's, is found in the transformed state. The simplified scheme can be realized by the following three steps (see fig.4).

i) The two qubits on Bob's side, $B$ and $C$, are input into a controlled-NOT gate, where $C$ is the controller. Then Bob makes a $\sigma_z$ measurement on $A$ and sends the result to Alice via one cbit classical communication. They would perform bit-flip rotation $\sigma_z$ on $A$ and $C$ respectively for result $|0\rangle$, while doing no operation for $|1\rangle$. The remaining state $A$ and $C$ is $|\Psi\rangle_{AB} = \alpha |0\rangle |0\rangle + \beta |1\rangle |1\rangle$.

ii) Alice implements the required operation on qubit $A$. 
$$U_{\text{com}}(\varphi) \otimes I_B |\Psi\rangle_{AB} = \alpha e^{i\varphi} |0\rangle |0\rangle + \beta e^{-i\varphi} |1\rangle |1\rangle$$
iii) A measurement is performed on $A$ and the result is sent to Bob. $B$ is
found in the desired state $|\psi'\rangle_B = U_{\text{com}}(\varphi) |\psi\rangle_B$ for the result $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ or $|\psi''\rangle_B = \sigma_z U_{\text{com}}(\varphi) |\psi\rangle_B$ in for $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. The latter can be converted into by an additional $\sigma_z$ rotation.

The successful probability is 100% and the total resources needed are one ebit shared and one cbit in each direction.

4 Effects of Noisy Entanglement

All the discussions above are based on the maximally entangled states shared between Alice and Bob. because of the interaction between the system and the environment, the entanglement will be partially destroyed. Such decoherence is possible during the distribution or storage of entanglement. In this section we discuss the effect of noisy entanglement to RSP by using the fidelity.

Generally, all physical processes, including decoherence evolution, can be represented by a complete positive map. Suppose Alice and Bob share an entangled state after decoherence,

$$\rho_{AB} = \tilde{S} (|\Psi^-\rangle_{AB} \langle\Psi^-|) , \quad (11)$$

where $\tilde{S}$ is the operator of decoherence evolution. The final state that Bob obtains after the five steps (1 – 5) above will be

$$\rho_B = \frac{Tr_{A_b}[(|1\rangle_A \langle 1| \varrho_{ABA})]}{Tr_{ABA}[(|1\rangle_A \langle 1| \varrho_{ABA})]} , \quad (12)$$

where

$$\varrho_{ABA} = U_A(r) \otimes U_{\text{CNOT}}(A: a) \otimes U_A(\theta, \phi)^\dagger \rho_{AB} U_A(\theta, \phi) \otimes U_{\text{CNOT}}(A: a)^\dagger \otimes U_A(r)^\dagger . \quad (13)$$

The effects of decoherence on RSP can be denoted by the state fidelity between $\rho_B$ and $\rho(\overrightarrow{T})$ (to be prepared),

$$F(\rho(\overrightarrow{T}), \rho_B) = Tr[\sqrt{\rho(\overrightarrow{T})^{1/2} \rho_B \rho(\overrightarrow{T})^{1/2}}] . \quad (14)$$

Here, two types of decoherence, dephasing and depolarizing, are considered.
First, for dephasing decoherence,\[\rho'_{AB}(p) = p |\Psi^-(\rangle_{AB} \langle \Psi^-| + \frac{1-p}{2} (|0\rangle_A \langle 0| \otimes |1\rangle_B \langle 1| + |1\rangle_A \langle 1| \otimes |0\rangle_B \langle 0|)\]

(15)
can be considered as a mixture of maximal entanglement and classical correlation. The results are\[\rho''_B = \frac{1+p}{2} \left( \frac{1+r}{2} |\psi_\perp\rangle \langle \psi_\perp| + \frac{1-r}{2} |\psi_\perp\rangle \langle \psi_\perp| \right) + \frac{1-p}{2} \left( \frac{1+r}{2} |\psi'_\perp\rangle \langle \psi'_\perp| + \frac{1-r}{2} |\psi'_\perp\rangle \langle \psi'_\perp| \right)\]

(16)
and
\[F(\rho(r), \rho''_B) = \sqrt{\left( \frac{\alpha + \beta}{2} \right)^2 + \left( \frac{\alpha - \beta}{2} \right)^2 + \gamma^2} - \sqrt{\left( \frac{\alpha + \beta}{2} \right)^2 + \gamma^2}\]

(17)
where\[|\psi_\perp\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle, |\psi'_\perp\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle,\]
\[\alpha = \frac{1}{8} \left( (1+p)(1+r)^2 + (1-p)(1+r)(1+r \cos 2\theta) \right)\]
\[\beta = \frac{1}{8} \left( (1+p)(1-r)^2 + (1-p)(1-r)(1-r \cos 2\theta) \right)\]
\[\gamma = \frac{1}{8} r(1-p) \sqrt{1-r^2 \sin 2\theta}.\]
Specially, for maximal entanglement shared \(p = 1, F = 1\); for pure states \(r = 1, F = \frac{1}{2} \sqrt{3} + p + (1-p) \cos 2\theta\); and for \(r = 0, F = 1\).

The entangled state after depolarizing is\[\rho''_{AB}(p) = p |\Psi^-(\rangle_{AB} \langle \Psi^-| + (1-p) \frac{I_A}{2} \otimes \frac{I_B}{2}\]

(18)
which can be considered as a mixture of maximal entanglement and maximal mixed state. The result state that Bob gets is\[\rho''_B = \frac{1 + pr}{2} |\psi\rangle \langle \psi| + \frac{1 - pr}{2} |\psi_\perp\rangle \langle \psi_\perp|\]

(19)
and the state fidelity is\[F(\rho''(r), \rho''_B) = \frac{1}{2} \left( \sqrt{(1+r)(1+pr)} + \sqrt{(1-r)(1-pr)} \right)\]

(20)
Specially, for maximal entanglement shared \(p = 1, F = 1\); for pure states to be prepared \(r = 1,F = \sqrt{2}\); and for maximally-mixed states \(r = 0, F = 1\).

Here we found that the fidelity for depolarizing is found to be independent on states to be prepared. While for dephasing, it depends only on the proportion of \(|0\rangle\) and \(|1\rangle\), i.e. depending on \(\theta\), but not on the relative phase \(\phi\). And, obviously, the later is better than former, which means classical correlation can help remote state preparation.
5 Experiments of Remote State Preparation for Single Photons

We have given the scheme of remote state preparation and quantum remote operation theoretically and analyze the effects of noisy entanglement to RSP. In this section, we carry out an experiment of remote state preparation for single photons by using spontaneous parametric down conversion (SPDC) and linear optical elements. Here, we only discuss the RSP in dephasing noisy channel. The setup is represented in Fig. 5. A pulse of ultraviolet (UV) light pass through a BBO crystal (0.5 mm, cut for type-II phase match). The UV pulse is frequency-doubled pulse (less than 200fs with 82MHz repetition and 390nm center-wavelength) from a mode-locked Ti: sapphire laser (Tsunami by Spectra-Physics). Because of the birefringence of ordinary light (o light) and especial light (e light) in BBO crystal, the state of the biphoton from SPDC process is no longer the maximal entangled one, but is the state like

$$\rho_{AB}(p) = p|\Psi^-\rangle_{AB}\langle\Psi^-| + \frac{1 - p}{2}(|H\rangle_A\langle H| \otimes |V\rangle_B\langle V| + |V\rangle_A\langle V| \otimes |H\rangle_B\langle H|).$$  

(21)

Where $p$ can be adjusted by quartz waveplate. In our experiment, we only chose $p = 0.9$ and 0.7.

In Fig. 4, photon A passes through path 2. First, it is rotated by QWP1 and HWP1 which realize the unitary operator $U_A(\theta, \phi)^\dagger$. Second, photon A transmits a birefringent crystal (3.0 \(\beta\)-BBO). Here we chose the time of photon A passing through BBO crystal to be the auxiliary qubit ($|t_o\rangle$ for ordinary light and $|t_e\rangle$ for extra-ordinary light). A CNOT operation will be accomplished after the photon A passes through the BBO crystal.
\[(a|H\rangle + b|V\rangle)|t_0\rangle \rightarrow a|H\rangle|t_o\rangle + b|V\rangle|t_e\rangle.\]

After 3.0 mm BBO crystal, the separation between e\((H)\)- and o\((V)\)-polarized light is about 300fs. Because the coherent time of the wavepacket is about 200fs, the e\((H)\)- and o\((V)\)-polarized light is no longer coherent superposed. Following 3.0 mm BBO, photon A is operated by HWP2 which realizes the unitary \(U_A(r)\). The time qubit is traced after photon is detected by a single photon detector. Photon B passes through path 1, and quantum state tomography is performed to reconstruct the state matrix of it.

In our experiments, several pre-agreed state sets are selected (see Fig 1). Three set of pure states: 1)the polar great circle on the sphere crossed by X-Z plane \((\theta \in [0, \pi], \phi = 0)\), 2)the polar great circle by Y-Z plane \((\theta \in [0, \pi], \phi = \pi/2)\), and 3)the equatorial great circle by X-Y plane \((\theta = \pi/2, \phi \in [0, 2\pi])\), see Fig 6. It is found that the fidelity is independent of the relative phase \(\phi\), as eq.(1) shows. Four sets of mixed states are shown in Fig. 7 and 8: 4)a small circle on the X-Z plane \((r = \cos^2 \frac{\pi}{8}, \theta \in [0, \pi], \phi = 0)\); 5)the zero vector \(\overrightarrow{O}\); and two lines on the X-Z plane: 6) \((r \in [-1, 1], \theta = \pi/4, \phi = 0)[19]\) and 7) \((r \in [-1, 1], \theta = \pi/2, \phi = 0)[19]\). It is found that all experiment results (the square dots) agree with the theoretical prediction (the solid lines) well.

Figure 6: Fidelity of pure states from different ensembles: 1) In Fig. 4a and fig. 4b, the polar greatest circle on the sphere in X-Z plane; 2) In Fig. 4c and Fig. 4d, the polar greatest circle on the sphere in Y-Z plane; 3) In Fig. 4e and Fig. 4f, the states on the equatorial greatest circle.
A Scheme of Remote Operation

In this section, we present a experimental scheme of quantum remote operation. The experimental setup is shown in fig. 9. The UV light is injected in the Kwiat-type BBO-pair to generate the polarization-entangled biphoton state.

1') The photon pair from SPDC is polarization-entangled.

\[ |\Phi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B) \]

A polarization beam splitter followed by an half-wave-plate at 45° in one path is used to convert the state into the state entangled between the polarization of A and the path of B.

\[ |\Phi'\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |u\rangle_B + |V\rangle_A |d\rangle_B) |H\rangle_B \]

And the polarization of photon B can be prepared into arbitrary state. Here one HWP and one QWP in each path are used to prepare an arbitrary pure state of polarization.

\[ |\Phi'\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |u\rangle_B + |V\rangle_A |d\rangle_B) (\alpha |H\rangle_B + \beta |V\rangle_B) \]

2') Another polarization beam splitter PBS2 is used as a controlled-NOT gate of qubit.

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (\alpha |H\rangle_A |H\rangle_B |u\rangle_B + \beta |V\rangle_A |V\rangle_B |u\rangle_B + \alpha |V\rangle_A |H\rangle_B |d\rangle_B + \beta |H\rangle_A |V\rangle_B |d\rangle_B) \]
Figure 8: Fidelity of mixed states from different ensembles. The states in the above and below two figures are from the two lines in X-Z plane: (r2208[-1,1], =/4, =0) and (r2208[-1,1],=/2,=0) respectively.

![Figure 8](image)

Figure 9: The experimental setup of remote operation

The path qubit is measured at the basis \{\ket{u}, \ket{d}\} by Bob. For result \ket{d}, a \sigma_x operation should be performed on the polarization qubit of particle A.

\[ |\Psi'\rangle = \alpha |H\rangle_A |H\rangle_B + \beta |V\rangle_A |V\rangle_B \]

3') The operation \( U_{\text{com}} \)

\[ U_{\text{com}} = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix} \]

is implemented on photon A by Alice, using two QWPs and one HWP between them[20].

4') Alice perform an Hadama operation on A followed by a \sigma_z measurement. After receiving Alice’s measurement result, Bob would perform a \sigma_z operation.
on B for result |V⟩, or do nothing for |H⟩. The final polarization state of B is just

7 Conclusion

We have present a practical and general scheme of remote preparation for pure and mixed states. An auxiliary qubit and controlled-NOT operation are used in the scheme. The effects of noisy entanglement are discussed for two important types of decoherence, depolarizing and dephasing, by the state fidelity. The fidelity for depolarizing is found to be independent on states to be prepared. While for dephasing, it depends only on the proportion of |0⟩and |1⟩, i.e. depending on θ, but not on the relative phase φ. And the dephasing entanglement is always better than the depolarizing one for RSP, which implies classical correlation is helpful for RSP. In our experiment, we successfully complete RSP of pure and mixed states via dephasing entanglement by using spontaneous parametric down conversion (SPDC) and linear optical elements.

We have given a modified protocol scheme of quantum remote operation. Here, the set operation is restricted to either $U_\text{com}$ or $U_\text{anti}$. The resource to complete the quantum remote operation is only one ebit shared and one cbit in each direction. And a scheme is also presented, which is practical and possible for realization in laboratory.

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References


[19] Here we write \( r \in [0, 1], \theta = \pi/2, \phi = 0 \) \( \cup \) \( r \in [0, 1], \theta = \pi/2, \phi = 0 \) as \( r \in [-1, 1], \theta = \pi/2, \phi = 0 \) and write \( r \in [0, 1], \theta = \pi/4, \phi = 0 \) \( \cup \) \( r \in [0, 1], \theta = \pi/2, \phi = 0 \) as \( r \in [-1, 1], \theta = \pi/4, \phi = 0 \) for the convenience.