Optimization methods of tilted multi-focus Fresnel binary optical elements

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ABSTRACT

In the Fresnel condition, the design and optimization method of binary optical element with multi-focus has been investigated. A one-step optimizing algorithm of tilted multi-focus diffractive optical element is presented by using a tilted image plane in the Fresnel approximation. Based on iterative Fourier transform algorithm (IFTA), not only the higher diffraction efficiency, increased about four percent, but also the instability of distributing at the different focus, namely different tilted angle, are obviously improved by the modified simulated annealing algorithm (SA). Comparison on one-step and multi-step circle algorithm of optimizing multi-focus diffraction element shows both the faster speed and the higher diffraction efficiency for the one-step algorithm.

Key words: binary optics, diffraction, iterative Fourier transform algorithm, simulated annealing algorithm, tilted multi-focus

1 INTRODUCTION

Binary optical elements are currently used in the beam splitting and beam shaping, i.e. the transformation of a light beam with certain characteristics into one with a desired spatial phase or amplitude distribution or both, and they are very important elements of multi-function, lightweight optical systems. Designing and optimizing methods for diffractive optical elements of large depth of focus and multi-focus image at different lengthways distance have been developed.

In the Fraunhofer approximation binary optical elements are focused in infinity. In many applications, however, binary optical elements need be focused in finity. Furthermore, in special cases, for instrument structures and detectors needs multi-focus diffractive optical elements are designed, which the object images of different focus plane are optimized by multi-step circle algorithm respectively. Then phase structures are combined after iterative algorithm. Finally, phase distributing of multi-focus diffractive optical elements is obtained. In focus plane, diffraction efficiency of single focus is 10.7% that is three focuses of two phase levels. The process of algorithm is excessive and diffraction efficiency of phase structures is low. Based on the Kirchhoff diffractive formula, we can universality get optimization algorithms of diffractive optical elements in the Fresnel approximation. Far field Fourier iterative algorithm and near field Fresnel iterative algorithm are only special cases of tilted multi-focus Fresnel diffractive formula. Multi-focus diffractive optical elements of pure phase are optimized by one-step using tilted focus plane. Discussions of optimization parameters show that diffraction efficiency can increase 4% about using improved simulated annealing algorithm and stability of optimization result is obviously improved in the different focus. Comparison on one-step and multi-step circle algorithm of optimizing multi-focus diffraction element shows both the
faster speed and the higher diffraction efficiency for the one-step algorithm.

2 ANALYZE THEORY WITH FRESNEL DIFFRACTIVE OF TILTED FOCUS PLANE

A schematic drawing is shown in Fig. 1, where aperture plane is situated in the \( x_0y_0 \) plane and observed plane is situated in the \( xy \) plane and \( \alpha \) is angle between them. Based on the Kirchhoff diffractive formula, tilted multi-focus Fresnel diffractive formula is presented as follows.

In Fig. 1, the figure of diffractive space and observed plane

In Fig. 1, the distance from \( Q \) in the \( x_1y_1z_1 \) reference frame to \( P \) in arbitrary dot of the aperture plane may be expressed by

\[
r = \left[ (z_1 + d)^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2 \right]^{1/2}
\]

The relation between observed \( xy \) plane and the \( x_1y_1z_1 \) reference frame may be considered as revolving transform that is

\[
x_1 = x \cos \alpha - z \sin \alpha, \\
y_1 = y, \\
z_1 = x \sin \alpha + z \cos \alpha,
\]

which is taken into expression (1). In the Fresnel approximation, pulse response of the system can be expressed by

\[
h(x_0 - y_0; x - y) = \frac{\exp(jkd) \exp(jkx \sin \alpha) \exp(jkz \cos \alpha) \exp(jk \frac{x^2 + z^2}{2d})}{j \lambda d} \times
\]

\[
\exp\left\{ \frac{k}{2d} [-2xx_0 \cos \alpha + 2zx_0 \sin \alpha + x_0^2 + (y - y_0)^2] \right\}
\]

and taken expression (2) into diffractive cumulation integral then

\[
G(x, y, z) = \frac{\exp(jkd) \exp(jkx \sin \alpha) \exp(jkz \cos \alpha) \exp(jk \frac{x^2 + y^2 + z^2}{2d})}{j \lambda d} \times
\]

\[
\int \int_{-\infty}^{\infty} U(x_0, y_0) \exp\left\{ \frac{k}{2d} [-2xx_0 \cos \alpha + 2zx_0 \sin \alpha + x_0^2 - 2yy_0 + y_0^2] \right\} dx_0 dy_0
\]
Variable transform  \( x_i = x \cos \alpha \) then expression (3) may be become

\[
G(\frac{x}{\cos \alpha}, y, z) = \frac{\exp(jkd) \exp(jkx \tan \alpha) \exp(jkz \cos \alpha) \exp(\frac{x^2 + y^2 + z^2}{2d})}{j\lambda d \cos \alpha} \times \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x_0, y_0) \exp[j\frac{\pi}{\lambda d}(x_0^2 + y_0^2)] \exp[j\frac{2\pi}{\lambda d} z(x_0 \sin \alpha)] \exp[-j\frac{2\pi}{\lambda d} (xx_0 + yy_0)] dx_0 dy_0
\]

(4)

If aperture plane parallels with observed plane, that is \( \alpha = 0 \), expression (4) will be known to Fresnel diffractive formula. If d is long distance satisfied with the Fraunhofer approximation, the Fraunhofer diffractive formula can be obtained.

3 OPTIMIZATION ALGORITHM AND DESIGN OF DIFFRACTIVE OPTICAL ELEMENT WITH FUNCTION OF TILTED MULTI-FOCUS

Iterative Fourier transform algorithm is ofen used for pure phase diffractive optical elements. The procedure that designing and optimizing method is used for tilted multi-focus Fresnel diffractive optical elements is presented as follows. Diffractive optical element is placed in the \( x_0y_0 \) plane shown in fig.1. The object image is put in the xy plane, i.e. recovery image appears in the tilted plane. The phase distributing of diffractive optical plane is expressed by quadratic phase coefficient \( G(x_0, y_0, z_0) = \text{RAND}(x_0, y_0, z_0) \). Based on expression (4), Fourier transform of \( G(x_0, y_0, z_0) \) is calculated, omitted constant phase coefficient before integral expression, then

\[
U'(\frac{x}{\cos \alpha}, y, z) = FT\left\{ G(x_0, y_0, z_0) \exp[j\frac{2\pi}{\lambda d}(x_0^2 + y_0^2)] \exp[j\frac{2\pi}{\lambda d} z(x_0 \sin \alpha)] \right\}
\]

(5)

where \( U'(\frac{x}{\cos \alpha}, y, z) \) is optical field distributing in the tilted plane. The \( U(\frac{x}{\cos \alpha}, y, z) \) of object optical field is calculated by inverse Fourier transform substituting the \( U'(\frac{x}{\cos \alpha}, y, z) \). The new \( G(x_0, y_0, z_0) \) is obtained by \( U(\frac{x}{\cos \alpha}, y, z) \). The amplitude of optical field \( G(x_0, y_0, z_0) \) is constrained with constant amplitude, and pure phase distributing is obtained in the diffractive plane. After the iteration many times, the iteration will stop when calculated outputs are converging on a set of relative stable values. In fig.2, the schematic drawing of IFTA of tilted multi-focus is given.
Diffraction efficiency of recovery image is primary optimizing parameter showing distinction between recovery image and object image, which is defined by

\[ \eta = \frac{\sum_{(x,y) \in \Omega} I_r(x,y)}{\sum_{x}^{M} \sum_{y}^{N} I_r(x,y)} \]  

where \( \Omega \) is signal windows sizes, and M and N are signal matrix sizes (unit is pixel).

We now employ the above algorithm to design a diffractive optical element, which can focus three focuses in the lengthways tilted plane. Design parameter: signal matrix sizes 256 × 256 pixels, signal window size one pixel, elementary pixel size 2.5\( \mu \)m, the distance from the plane of diffractive optical element to the center of tilted plane is 600\( \mu \)m, tilted angle is 30 degrees, three focuses of tilted plane are situated in line, the distance from middle focus to up focus is equal from middle focus to low focus, where is 200\( \mu \)m, and desired focal lengths 500\( \mu \)m, 600\( \mu \)m, and 700\( \mu \)m. When choosing \( \lambda \) = 650nm and 50 IFTA iterations, diffraction efficiency is 38.13% at 2 phase levels and 76.65% at 4 phase levels and 89.68% at 8 phase levels. Furthermore, when detector parallels with diffractive optical element, diffraction efficiency of three focuses is 10.76%, 16.52% and 10.85% respectively at 2 phase levels. Fig. 3(a) is phase distributing of tilted multi-focus diffractive optical element, (b) is its recovery image, and (c) is phase distributing of multi-step diffractive optical element.

Fig. 3 The optimization result of multi-focus diffractive element
With the same structure parameters, optimizing result is compared between one-step optimization algorithm of tilted multi-focus and multi-step iteration algorithm of multi-focus DOE. The calculating result is shown in Tab. 1, where \( \eta_1 , \eta_2 \) and \( \eta_3 \) are expressed as diffraction efficiency of top, middle and low focus respectively, and \( t \) is optimizing time. In the same condition, optimization algorithm of tilted multi-focus may be easily achieved and have greater diffraction efficiency, which is shown in Tab. 1. Although multi-focus DOE may be designed and 4 phase levels and 8 phase levels may be optimized by multi-step optimization algorithm, which diffraction efficiency is lower and optimizing time is longer.

Tab. 1 The comparison result of two algorithms

<table>
<thead>
<tr>
<th>Evaluating parameters</th>
<th>( \eta_1 )</th>
<th>( \eta_2 )</th>
<th>( \eta_3 )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-step multi-focus optimization (2 phase levels)</td>
<td>10.84%</td>
<td>10.65%</td>
<td>10.87%</td>
<td>63.366s</td>
</tr>
<tr>
<td>Tilted multi-focus optimization (2 phase levels)</td>
<td>10.76%</td>
<td>16.52%</td>
<td>10.85%</td>
<td>27.035s</td>
</tr>
<tr>
<td>Multi-step multi-focus optimization (4 phase levels)</td>
<td>21.93%</td>
<td>21.52%</td>
<td>21.68%</td>
<td>64.620s</td>
</tr>
<tr>
<td>Tilted multi-focus optimization (4 phase levels)</td>
<td>21.67%</td>
<td>33.39%</td>
<td>21.59%</td>
<td>28.369s</td>
</tr>
<tr>
<td>Multi-step multi-focus optimization (8 phase levels)</td>
<td>25.43%</td>
<td>24.91%</td>
<td>25.61%</td>
<td>66.297s</td>
</tr>
<tr>
<td>Tilted multi-focus optimization (8 phase levels)</td>
<td>25.48%</td>
<td>38.8%</td>
<td>25.41%</td>
<td>29.517s</td>
</tr>
</tbody>
</table>

For the sake of evaluating the characteristic of algorithm, when tilted angle changes from 20 degrees to 70 degrees, i.e. transverse distance becomes large between two focuses, a curve of diffraction efficiency is shown in Fig. 4 with IFTA optimization algorithm. We can know that diffraction efficiency will fluctuate with transverse distance becoming large between two focuses. It may know that we only adopt IFTA not to obtain stable optimization result. So the algorithm need be improved to get stability of optimization result. To avoid limitation that optimization result depends on tilted angle with IFTA, we adopt modified SA to optimize tilted multi-focus DOE. The intermediate solution \( G_{\text{int}} \) of the optimization result with IFTA is subjected to the procedure outlined in Fig. 5. The part phase distributing is modified at random. And a new solution \( G_{\text{mod}} \) is used as the start distribution for another optimization through the IFTA. If diffraction efficiency is increased through IFTA, the better \( G_{\text{end}} \) is memorized. The algorithm will go on until converging on a set of relative stable values.
Diffraction efficiency, which is shown in Fig. 6 where the real line indicates optimization result of IFTA and the dotted line indicates optimization result of SA, is compared between IFTA optimization algorithm and SA optimization algorithm. In Fig. 6, we can know that not only diffraction efficiency is increased about four percent but also optimization result of diffraction efficiency is stable and influence on optimization result of different tilted angle is lesser.
5 CONCLUSION

We present one-step optimization method of lengthways multi-focus DOE in the Fresnel approximation, which is the most universal optimization method of binary optical elements. Not only diffraction efficiency is increased but also stability of optimization result is better using the improved SA. The one-step optimization method of multi-focus DOE not only optimization speed is faster but also diffraction efficiency is higher. The mentioned method may be directed to design binary optical elements in the Fresnel approximation.

REFERENCES


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