Design and quantitative analysis of diffractive optical element to realize true beam smoothing on an inclined plane

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ABSTRACT

Diffractive optical element (DOE) is a proposing technique to realize beam smoothing in inertial confinement fusion (ICF) system. In indirect-driving ICF system, beam smoothing is required on a certain inclined plane non-perpendicular to the optical axis. In this paper, the DOE is designed to be consistent with beam smoothing with long focal depth, which ensures that the beam smoothing can be obtained on the inclined plane. And in the optimization, the sampling interval is chosen as half of the traditional sampling interval, in order to realize true beam smoothing distribution. The simulated results show that the performance of the beam smoothing can be held with any sampling interval on the inclined plane. Finally, the performance of the DOE is quantitatively analyzed. Two parameters, light efficiency and top non-uniformity, are re-defined. These two parameters can truly and exactly evaluate the performance of the DOE, because their definitions can go back to the origin. The simulated results show the validity of the design with long focal depth and half of the traditional sampling interval.

Keywords: diffractive optical element, true beam smoothing, inclined plane, long focal depth

1. INTRODUCTION

Beam smoothing with uniform intensity distribution is required in many kinds of applications, including inertial confinement fusion (ICF), laser manufacturing and so on. Specially, in indirect-ICF system, beam smoothing should be obtained on the hohlraum, which is non-perpendicular to the optical axis [1]. Diffractive optical element (DOE) has many advantages in realizing such beam smoothing, including high light efficiency, high flexibility in phase design and so on. Therefore, DOE’s application in ICF system to realize beam smoothing is focused on since 1994 [2]. Many optimization algorithms [2,3,4,5] have been proposed and adopted to design DOE to obtain beam smoothing on the focal plane, where Fourier Transform optically performs. Good simulated results have been obtained by using these algorithms. Generally speaking, the sampling interval is consistent with the sampling theorem and chosen as \( \Delta = \frac{\lambda f}{D} \), where \( f \), \( \lambda \) and \( D \) are the focal length of the Fourier Transform lens, the incident wavelength and the aperture size of the DOE, respectively.

Unfortunately, the beam smoothing on the focal plane optimized with such a sampling interval is not true beam smoothing. Only the sampling points in the optimization are consistent with the required beam smoothing, the other points are apart from such a requirement. When the sampling interval is chosen as \( \Delta = \frac{\lambda f}{D/2} \), the true beam smoothing can be obtained after the optimization [6]. The intensity of any point on the focal plane will be consistent with the required beam smoothing.

To obtain true beam smoothing on a certain inclined plane, an indirect design method is applied. The DOE is designed to be consistent with true beam smoothing with long focal depth, which is realized by modifying the phase of the DOE to obtain true beam smoothing on three selected planes at right angles with the optical axis. Then the true beam smoothing can be realized on the inclined plane.

The performance parameters calculated with the chosen sampling points are not sufficient to physically and...
quantitatively evaluate the performance of the DOE, because the measure of the set of the chosen sampling points is zero. Based on the spatial frequency spectrum, the performance of the DOE is quantitatively analyzed. Two parameters, light efficiency and top non-uniformity, are re-defined. These two parameters can truly and exactly evaluate the performance of the DOE, because their definitions can go back to the origin. Finally, the performances of the DOE are re-calculated, and the calculated results show the validity of the design with long focal depth and half of the traditional sampling interval.

2. DESIGN PRINCIPLE

For simplicity, only one-dimensional problem is discussed. The designed DOE is a multi-phase level element and the width of the phase cell is considered, then the transmittance of the DOE is

\[ T(x) = \sum_{j=1}^{N} \exp(i\varphi_j) \text{rect} \left( \frac{x - (2j - (N + 1))D/(2N)}{D/N} \right), \]  

where \( x \) is the coordinate of the near field, \( \varphi_j \) is the phase value of the \( j \)-th cell, \( N \) is the number of the phase cell, \( D \) is the aperture size of the DOE, and

\[ \text{rect}(x) = \begin{cases} 
1 & \text{when } |x| \leq \frac{1}{2} \\
0 & \text{else} 
\end{cases}. \]  

Let \( a = D/N \), when a plane wave is incident, according to Kirchhoff theory, the intensity distribution on the focal plane can be written as

\[ I(x', f) = \left| \sum_{j=1}^{N} \exp(i\varphi_j)a \sin\left(\frac{a x'}{\lambda f}\right) \exp\left(-i \frac{2\pi(2j - 1)}{2} \frac{a x'}{\lambda f}\right) \right|^2, \]  

where \( x' \) is the coordinate of the focal plane, and

\[ \sin(c(x)) = \begin{cases} 
1 & \text{when } x = 0 \\
\sin(\pi x) / (\pi x) & \text{else} 
\end{cases}. \]  

The intensity distribution on a plane at right angle with the optical axis and with defocusing amount \( \Delta f \) is

\[ I(x, f + \Delta f) = \left| \sum_{j=1}^{N} \exp(i\varphi_j) \exp(-\frac{i\pi j - (N + 1) a i}{2} \frac{\Delta f}{f(f + \Delta f)})a \sin\left(\frac{a x}{\lambda f}\right) \exp\left(-i 2\pi(2j - 1) \frac{a x}{2 \lambda f}\right) \right|^2. \]  

In optimization, only the intensity values of some discrete sampling points can be computed. To realize true beam smoothing, the sampling interval in Equ. (3) and (5) should be chosen as \( \Delta = \lambda f / D / 2 \).

To realize true beam smoothing on an inclined plane, three selected planes at right angles with the optical axis are involved. Ignoring the constant \( a \), the phase factor and the envelope of sinc function, the intensity values with \( \Delta = \lambda f / D / 2 \) on these three planes can be expressed as

\[ I(n, f) = \left| \sum_{j=1}^{N} \exp(i\varphi_j) \exp(-i\frac{2\pi nj}{2N}) \right|^2. \]
\[ I(n, f + \Delta f) = \left| \sum_{j=1}^{N} \exp(i \phi_j) \exp(\frac{-i \pi [(j - \frac{N+1}{2})d]^2}{\lambda f (f + \Delta f)}) \cdot \frac{\Delta f}{\lambda} \cdot \exp(-i 2 \pi \frac{n_j}{2N}) \right|^2, \] (7)

\[ I(n, f - \Delta f) = \left| \sum_{j=1}^{N} \exp(i \phi_j) \exp(\frac{-i \pi [(j - \frac{N+1}{2})d]^2}{\lambda f (f - \Delta f)}) \cdot \frac{-\Delta f}{\lambda} \cdot \exp(-i 2 \pi \frac{n_j}{2N}) \right|^2, \] (8)

where \( n = 0,1,2, \ldots, 2N-1 \).

### 3. SIMULATED RESULTS

Here, \( f, D, \lambda, d \) and \( \Delta f \) are 600 mm, 100 mm, 1.053 \( \mu \)m, 100 \( \mu \)m and 37.5 \( \mu \)m, respectively. With the sampling interval \( \Delta = \lambda f D/2 = 3.159 \mu \)m, the designed phase distribution of the DOE and the intensity distributions on three selected planes are shown in Fig. (1). The non-uniformities are 15.6%, 11.1% and 15.8%, respectively. The adopted hybrid algorithm and the definitions of the non-uniformity were described in detail in reference [5].

![Phase distribution of the DOE](image1.png)

![Intensity distribution with \( \Delta f = 37.5 \mu \)m](image2.png)

![Intensity distribution with \( \Delta f = 0 \mu \)m](image3.png)

![Intensity distribution with \( \Delta f = 7.5 \mu \)m](image4.png)

Fig. 1 The designed results with the sampling interval 3.159 \( \mu \)m.
On these three selected planes, when the sampling interval is chosen as 1 µm, the intensity distributions are shown in Fig.2, the non-uniformities are 17.6%, 12.6%, 16.0%, respectively. The non-uniformities change a little, which show true beam smoothing distributions are realized on these planes.

When an inclined plane at an angle of 53.13° with the optical axis is considered, the ideal uniform illuminated area has the size of 125 µm. The intensity distribution on this plane and with the sampling interval 0.75 µm is shown in Fig.3. The non-uniformity is 12.6%.

On this inclined plane, the relationship between the calculated non-uniformity and the chosen sampling interval is shown in Fig.4. The non-uniformity hardly changes and we can draw a conclusion that the inclined plane has the performance with true beam smoothing.

4. PERFORMANCERS ANALYSIS

When the inclined plane is at angle of θ to the optical axis, the intensity distribution on the inclined plane can be written as
\[
I(y) = \left| \sum_{j=1}^{N} \exp(i \varphi_j) \exp\left(-i\pi a^2 y \cos(\theta - \frac{N+1}{2}) \frac{\lambda f}{\lambda(f + y \cos \theta)} \right) \right|^2, \quad (9)
\]

where \( y \) is the coordinate on the inclined plane.

If \( f \gg y \cos \theta \), Equ. (9) can be rewritten as

\[
I(y) = \left| \sum_{j=1}^{N} \exp(i \varphi_j) \exp\left(-i\pi a^2 y \cos(\theta - \frac{N+1}{2}) \frac{\lambda f}{\lambda f^2} \right) \right|^2, \quad (10)
\]

where \( f(j) = -\frac{\pi a^2 \cos \theta}{\lambda f^2} (j - \frac{N+1}{2})^2 - \frac{2\pi \alpha y \sin \theta}{\lambda f} \).

Then

\[
I(y) = \sum_{j=1}^{N} \sum_{k=1}^{N} \cos[\varphi_j - \varphi_k + (f(j) - f(k))y] \]

\[
= N + 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} \cos[\varphi_j - \varphi_k + (f(j) - f(k))y] \quad . \quad (11)
\]

The intensity distribution on the inclined plane can be expressed as the sum of a series of cosine functions with different spatial frequency and initial phase.

Based on Equ.(11), the total intensity within the required area \( d_1 \leq y \leq d_2 \) is

\[
I = \int_{d_1}^{d_2} I(y)dy = N(d_2 - d_1)
\]

\[
+ 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} \frac{1}{f(j) - f(k)} \left[ \sin(\varphi_j - \varphi_k + (f(j) - f(k))d_2) - \sin(\varphi_j - \varphi_k + (f(j) - f(k))d_1) \right] \quad . \quad (12)
\]

The average intensity \( \bar{I} \) is

\[
\bar{I} = \frac{I}{(d_2 - d_1)} . \quad (13)
\]

The top non-uniformity can be re-defined as...
The total intensity is \(2\pi N^2\), and the light efficiency is re-defined as
\[
\eta = \frac{I \sin \theta}{(2\pi N^2)}.
\] (15)

The re-defined top non-uniformity and light efficiency can truly and exactly reflect the performance of the intensity distribution on the inclined plane, because their definitions can go back to the origin. The performance of the intensity distribution on the inclined plane at the angle of 53.13° is re-calculated, the top non-uniformity and the light efficiency are 15.3% and 87.4%, respectively. The top non-uniformities shown in Fig.4, calculated with several different sampling intervals on the inclined plane, is very near to the true value, which show the validity of the design with long focal depth and half of the traditional sampling interval.

5. CONCLUSION

A new design method with long focal depth and half of the traditional sampling interval has been proposed to design the DOE for realizing true beam smoothing on a certain inclined plane. The intensity distributions with several different kinds of sampling intervals on the inclined plane are consistent with the requirement of beam smoothing.

Finally, the performance of the DOE is quantitatively analyzed. Two parameters, light efficiency and top non-uniformity, are re-defined. These two parameters can truly and exactly evaluate the performance of the DOE. The simulated results show the validity of the design with long focal depth and half of the traditional sampling interval.

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