Electromagnetic analysis of diffractive lens with C method and local linear grating model

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ABSTRACT

The electromagnetic theory should be applied to determine the diffraction efficiency of structures whose minimum line width is comparable with wavelength or the grooves are too deep, where scalar theory is no longer useful. The coordinate transformation method (the C method) is a very efficient method for obtaining continuous surface-relief grating efficiency for both TE and TM polarization. The local linear grating model (LLGM) models 2-D circular diffractive lens with combination of a series of local linear gratings. We synthesized and analyzed circular diffractive lens with a continuous profile not as previous authors who always use multi-lever structures. The result is compared with that of scalar theory and analysis using LLGM and rigorous coupled-wave theory. This optimization can be used as a complement of the scalar design of diffractive lens.

Keywords: Diffractive lens, C method, Local linear grating model

1. INTRODUCTION

Electromagnetic wave incident on a diffractive lens or zone plates converge because of diffraction not as ordinary lens that focuses because of refraction. The efficiency of diffractive lens can be valued with scalar theory when the wavelength \( \lambda \) and grating period \( d \) satisfying \( d > 10\lambda \) and the aspect ratio is not very high. Electromagnetic theory has to be used to analyze the efficiency of diffractive lens when one of the two conditions above is not satisfied. For this kind of diffractive lens the diffractive lens is divided into many local linear gratings, the efficiency can be obtained by applying electromagnetic theory of gratings on each local grating and summing up diffractive wave on focal points wanted. When analyzing or synthesizing local gratings, other authors focused on rectangle or multilevel gratings not continuous gratings. Synthesizing with multilevel structures is to approach some kind of continuous structure. In this paper we directly analyze and synthesize continuous local grating structure in the domain satisfying \( d < 10\lambda \). The diffractive field of each local grating is valued with the coordinate method (the C method) and we optimized the continuous relief local grating profiles to obtain the maximum diffraction efficiency. For simple computation complexity, we introduced gratings with one or two sinusoidal harmonics, and the higher harmonics are discarded. Sheng estimated the efficiency and synthesis the profile of dielectric diffraction lens with multilevel structure with local linear grating model, its essence is same with J.maser and E. Noponen. In this paper we still use the local linear grating model, but we synthesize the profile of diffraction lens with a niche genetic algorithm not as Sheng who used simulated annealing.

In section 2 the electromagnetic analysis method for diffractive lens is briefly reviewed. In section 3 the implement process is given including parametric details, and both the merits and the disadvantages of our optimization is discussed.

2. A CONCISE REVIEW OF ELECTROMAGNETIC ANALYSIS FOR DIFFRACTIVE LENS

The efficiency of circular diffractive lens can be analyzed as following steps:

First divide the diffractive lens into two regions, one is the center circular region in which the efficiency can be analyzed with scalar theory, the other is the outer annular region. Divide the annular region into many local linear gratings. Evaluate the diffracted wave field of the center circular region with scalar theory. Analyze the diffracted field of each local linear grating with electromagnetic theory of gratings. Sum up the total field on the focal points, and calculate the diffraction efficiency.
To maximize the main focus diffraction efficiency of the diffractive lens, the profile of the local gratings can be optimized as following two steps:

Step1: Maximize the following formula of each local linear grating.

\[ E = E_{TE} + E_{TM} \]  \hspace{1cm} (1)

\( E_{TE}, E_{TM} \) are the first order diffraction electric field of TE, TM polarization respectively.

Step2: After the first step, the phases of \( E_{TE}, E_{TM} \) are almost the same. The phases of \( E_{TE}, E_{TM} \) can be shifted to almost 0 by moving the local gratings radially according to detour phase.

To analyze efficiency of high order focuses should use corresponding high order diffraction electric field. The electromagnetic analysis method for diffractive lens can only estimate the diffractive efficiency because the outer annular region is divided into many local linear gratings and neglected the variable period of diffractive lens. And this method can lead to considerable degradation of wave-front quality.

3. IMPLEMENT AND DISCUSSION

A 2-D circular diffractive lens satisfying following geometric formula:

\[ r_m = (2mλf + (mλ)^2)^{1/2} \]  \hspace{1cm} (2)

\( m=1,2,3,..., f \) is the main focal length, and \( r_m \) is the radius of \( m \)th zone from the center.

The width or period of the \( m \)th zone is

\[ d_m \approx (λf) / r_m \]  \hspace{1cm} (3)

The center region with \( d_m \) larger than 10\( λ \) can be analyzed with scalar diffraction theory, and the outer region should be analyzed with electromagnetic theory.

Profile of a continuous grating can be written as:

\[ F(x) = \sum_{n=1}^{\infty} a_n \cos(nKx) + \sum_{n=2}^{\infty} b_n \sin(nKx) \]  \hspace{1cm} (4)

\( K=2\pi/d, \) \( d \) is the period of the grating. This formula has already considered the diffractive wave of gratings only have a phase shift after moving in the main plane, and will not produce any two that kind of gratings. It is equivalent to the following formula:

\[ F(x) = A_1 \sin(Kx) + \sum_{n=2}^{\infty} A_n \sin(nKx + \varphi_n) \]  \hspace{1cm} (5)

We use the C method to evaluate the diffraction wave of each local linear grating. The C method will not give the right answer when incident angle is 0. This can be solved by slightly changing the incident angle to a very small angle or using the reciprocity theorem. We used reciprocity theorem in our program. According to reciprocity theorem, the two first order diffraction field of fig. 1 should be the same if the incident wave is the same. The incident angel of fig. 1(b) is -arcsin(1/(n1*d)).

![Fig. 1. Schematic diagram of using the reciprocity theorem](image)

We first synthesize local gratings of a dielectric diffractive lens with sinusoidal profiles. The refractive index \( n \) of the lens is 1.46 and the wavelength \( λ \) is 0.6328 μm. The profile of the local gratings can be denoted from (5):
\[ F(x) = A_1 \sin(Kx) \]  

(6)

\[ A_1 \] is the only parameter to optimize. Then we synthesized local gratings with two sinusoidal harmonics, the local linear gratings can be denoted as:

\[ F(x) = A_1 \sin(Kx) + A_2 \sin(2Kx + \phi_2) \]  

(7)

There are three parameters to be optimized.

We use GA to optimize the parameters. The samples of GA is the population. It is a niche GA to keep the diversity of the population. In each generation, most of the computation time is spend on the solving eigenvalue problem of the direct problem of local gratings. The fitness function of each individual is from equation (1). The truncated order of the C method strongly depends on the depth of the grating, higher gratings need higher truncated order, and that means higher gratings need more computation time. Applying high truncated orders on shallow gratings will bring a great deal of unnecessary computation. So we apply higher truncated orders on high gratings and lower truncated orders on low gratings to save computing time. If two gratings have same depths, the algorithm is more difficult to converge and need higher truncated order. To make the optimization manipulable on a PC, the max grating height has to be restrained according to different grating periods. For example, we applied maximal height 8\( \lambda \) on grating period 2\( \lambda \) and 4.8\( \lambda \) on grating period 8\( \lambda \) respectively.

The initial population of GA is randomly constructed. We use binary code to encode coefficient \( A_n \) and phase \( \phi_n \). Better resolution can be obtained by introducing longer binary strings, but this is not free: the searching space will be bigger and need more computation. In our program the numerical precision of \( A_n \) and \( \phi_n \) is \( \lambda /10 \) and \( \pi /4 \) respectively, and we constrain the computation time of each local grating in 10 hours.

The selection operator is a tournament selection.

The crossover operator is one-point crossover at probability \( p_c = 0.6 \). The gene mutation probability is \( p_m \). We choose \( p_m = 1/(\text{the length of the binary string}) \).

Fig. 2 shows the first-order diffraction efficiency of the local gratings with a sinusoidal profile. The coordinate transformation method was applied to evaluate the first-order diffraction efficiency and phase. The efficiency falls fast because of the limitation of the C method mentioned before. It is hard to get efficiency higher than 50% even in a very low period range because the sinusoidal grating has a symmetrical structure.
Fig. 3 shows the first-order diffraction efficiency of local gratings with two sinusoidal harmonics. Fig. 4 and Fig. 5 show the first-order diffraction efficiency and phase of the local gratings with two sinusoidal harmonics of TE and TM polarization from 0 to $4\lambda$ respectively. Table 1 gives the optimized parameters of local gratings with two sinusoidal harmonics.

<table>
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<th>$d/\lambda$</th>
<th>1.2</th>
<th>1.4</th>
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<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
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<td>1.4</td>
<td>1.4</td>
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<tr>
<td>$\phi_1$</td>
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<td>0</td>
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<td>$\pi/4$</td>
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<tr>
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<td>0.7</td>
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The synthesized result of two harmonics is much better than that of one sinusoidal harmonic. Fig. 3 shows synthesized efficiency is high in a low grating period range, and the sharp drop of diffraction efficiency near $2\lambda$ of diffractive lens designed with scalar theory is easily avoided, but it drops rapidly at higher grating period range. And this drop can be saved in some extent by introducing higher order harmonics. Synthesizing with coupled-wave theory and 4 levers had received a much smoother efficiency curve than our result in region $d>3\lambda$. The original idea to introduce electromagnetic method in the region $d<10\lambda$ is to avoid the sharp drop near $d=2\lambda$ of the diffractive lens designed with scalar theory. Our method can be a complement of the scalar design in this regard though we have not obtained efficiency good enough in region $d>3\lambda$.

4. CONCLUSION

We gave the first continuous diffractive lens synthesizing result in this paper. The sharp drop of diffraction efficiency near $d=2\lambda$ is easily avoided after optimization. The result is good when the local linear grating period is low, but the diffraction efficiency drops quickly when the grating period is greater than $3\lambda$. This is because the C method is very efficient to treat shallow continuous gratings, but more difficult to converge for higher gratings. This method can be a complement of the scalar design.

In our future work, we’ll make the algorithm more efficient, and extend the algorithm to analyze the diffractive lens of finite height. And applying other electromagnetic analysis methods such as FMM in the range $3\lambda$ to $8\lambda$ may give a comprehensive solution of the optimization of continuous diffractive lens.

REFERENCES