Design of variable line-space plane gratings with holographic recording

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ABSTRACT

Variable Line-Space gratings play an important role in focusing and aberration-reduced. They are widely used in spatial spectrum and synchrotron radiation facilities. However the design and fabrication methods of them are difficult. The problem is to be able to record holographically the expected grooves distribution. The geometric theory of aspheric wave-front recording optics is briefly described. We introduce the genetic algorithm to the parameters optimization of holographic variable line-space gratings. In order to obtain a steady set of recording parameters, we propose that the objective function of the genetic algorithm should consider the effect of the recording parameters errors. The integral expression of the objective function is also derived to improve the efficiency of calculation. Design example of holographic variable line-space plane gratings for a position sensor is given to demonstrate the capability of this method. The line-profiles of variable line-space plane gratings with different recording parameters are also compared in this paper.

Keywords: holographic optics, variable line-space gratings, genetic algorithm, optical design, synchrotron radiation

1. INTRODUCTION

Variable line-space (VLS) gratings play an essential role in focusing and aberration-reduced. They are widely used in spatial spectrum and synchrotron radiation facilities and present interesting solutions to design high resolution monochromators. However the design and fabrication methods of them are difficult. The problem is to be able to record holographically the expected grooves distribution in order to take certain advantage of VLS gratings.

Geometrical optics has been shown to be adequate in the design of VLS gratings for common applications. Koike *et al.* [1] and Noda *et al.* [2] introduced spherical mirrors into the holographic recording system for the first time, this method can provide additional degrees of freedom in the construction of holographic gratings. Namioka *et al.* [3] derived explicit expressions of the groove parameters in considerable detail by following an exact ray tracing procedure analytically.

In this paper we briefly review the geometric theory of aspheric wave-front recording optics, then we introduce genetic algorithm to the parameters optimization of holographic VLS gratings. But the grooves distribution is sensitive to the recording parameters which are calculated by our procedure sometimes. In order to obtain a steady set of recording parameters, we propose that the objective function of the genetic algorithm should consider the effect of the recording parameters errors. The integral expression of the objective function is also derived to improve the efficiency of calculation. Design example of holographic variable line-space plane gratings for position sensor is given to demonstrate the capability of our method. We also compare the line-profiles of VLS plane gratings with different recording parameters in this paper.

2. METHOD OF DESIGN AND CALCULATION

2.1. Geometric theory of holographic recording system

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We refer to Fig. 1 to invest the relation between the groove pattern and the geometrical positions of the recording sources [3]. The optical system is consisting of two coherent point sources, C and D, two ellipsoidal mirrors, M_1 and M_2, and an ellipsoidal grating blank covered with photoresist, G. The wavelength of laser being used for the holographic recording is λ_0.

For convenience, we assume that:

(1) the difference of distance between $(CO_1 + O_1O)$ and $(DO_2 + O_2O)$ is an integer multiple of λ_0;

(2) the zeroth groove passes through O.

In common cases, we only care about the groove density along Y axis, and the grating blank G is a plane in general, so we can put $l = 0$ and $\xi = 0$, then the groove density along Y, namely, $n_r(w)$ can be expressed by:

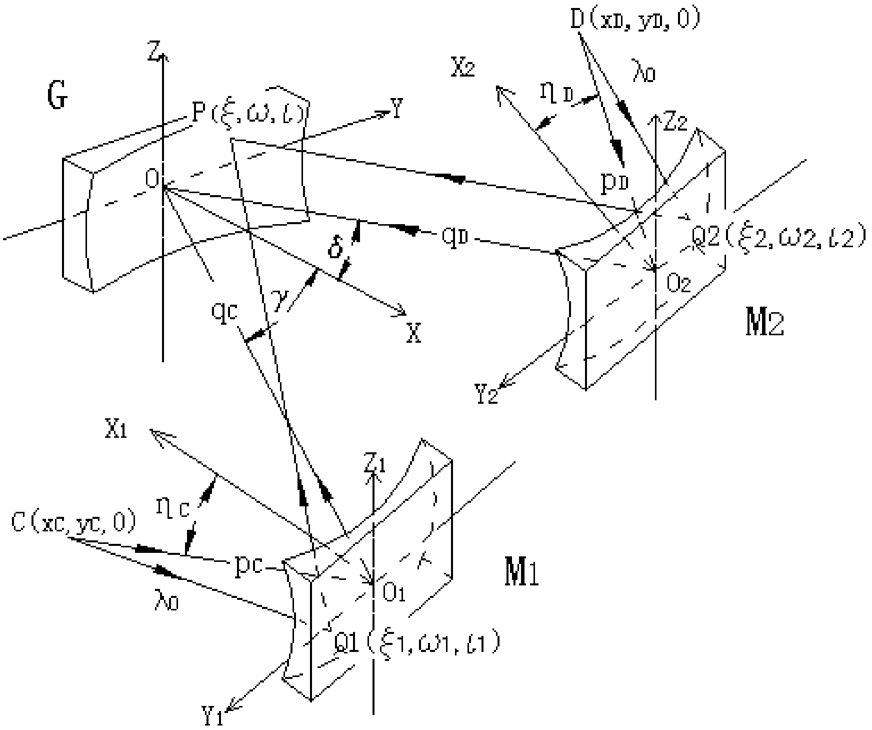

Fig. 1. Schematic diagram of an aspheric wave-front recording system

$$n_r(w) = \frac{1}{\lambda_0}[n_{10} + n_{20}w + \frac{3}{2}n_{30}w^2 + \frac{1}{2}n_{40}w^3] \quad (1)$$

In Eq. (1) n_{10}, n_{20}, n_{30}, and n_{40} is the coefficients of the groove density, they are determined by the recording parameters. Namioka *et al.* [3], Koike *et al.* [1] and Noda *et al.* [2] have derived analytical formulas for them in considerable detail.

2.2. Derivation of the objective function of the genetic algorithm

From discussion above we know that in the design of VLS gratings, there are a lot of recording parameters to be determined together with the semiaxes of M_1 and M_2 to get the expected groove density. But equations in reference 3 are very complex. In order to obtain optimum recording parameters, we use genetic algorithm (GA) in our program. Then we should establish the objective function first.

We assume that the expected groove density of a VLS plane grating $n_e(w)$ is following the law:

$$n_e(w) = n_0(1 + b_2w + b_3w^2 + b_4w^3) = n_0 + n_0b_2w + n_0b_3w^2 + n_0b_4w^3 \quad (2)$$

Let's select j points (including start and end points) with equal distance in the grating surface, and mark them with $w_i\,(i=1,2,...,j)$ in sequence from $w_{\min}$ to $w_{\max}$:

$$dw=\frac{(w_{\max}-w_{\min})}{(j-1)} \tag{3}$$

$$w_i = w_{\min}+(i-1)dw \tag{4}$$

The difference between $n_r(w_i)$ and $n_e(w_i)$, namely, $T(w_i)$ can be expressed by:

$$T(w_i)=n_r(w_i)-n_e(w_i)$$
$$=\left(\frac{n_{10}}{\lambda_0}-n_0\right)+\left(\frac{n_{20}}{\lambda_0}-n_0b_2\right)w_i+\left(\frac{3n_{30}}{2\lambda_0}-n_0b_3\right)w_i^{\;2}+\left(\frac{n_{40}}{2\lambda_0}-n_0b_4\right)w_i^{\;3}$$
$$=r_1+r_2w_i+r_3w_i^{\;2}+r_4w_i^{\;3}, \tag{5}$$

where:

$$r_1=\frac{n_{10}}{\lambda_0}-n_0 \tag{6a}$$

$$r_2=\frac{n_{20}}{\lambda_0}-n_0b_2 \tag{6b}$$

$$r_3=\frac{3n_{30}}{2\lambda_0}-n_0b_3 \tag{6c}$$

$$r_4=\frac{n_{40}}{2\lambda_0}-n_0b_4 \tag{6d}$$

They are functions of the recording parameters.

Let:

$$f=\frac{1}{j}\sum_{i=1}^{j}[T(w_i)]^2=\frac{1}{j}\sum_{i=1}^{j}[r_1+r_2w_i+r_3w_i^{\;2}+r_4w_i^{\;3}]^2 \tag{7}$$

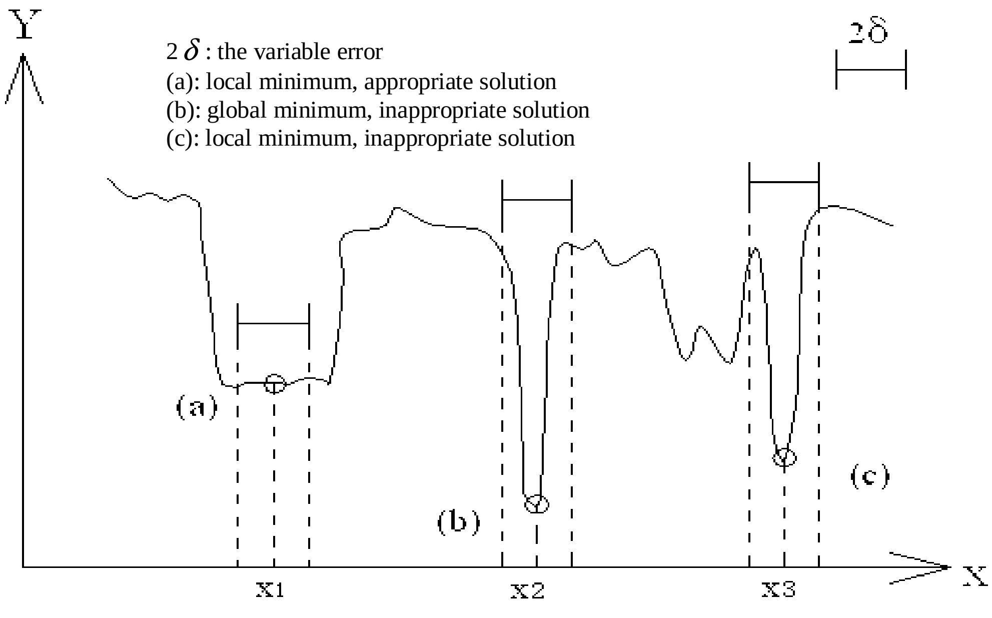

Fig. 2. Schematic diagram of the effect of the variable error

Even if the recording parameters have been accurately adjusted in experiment, they will change because of the excursion of experiment environment, such as the vibration, temperature and humidity. Then the groove parameters will change also. It's necessary for us to consider the excursion of the recording parameters in the optimization program.

For convenience, we consider a function Y=f(x) to investigate the effect of the variable error (see Fig. 2). We assume that the function image is as shown in Fig. 2. From Fig. 2 we know that points (a) and (c) is local minimum and point (b) is global minimum of the function. The function value maintain stabilization near point (a) and change quickly when the variable x changed near points (b) and (c), so point (a) is the appropriate solution to Min(Y).

In order to obtain a steady set of recording parameters, we introduce error function σ_f to evaluate the f function's sensitivity to recording errors:

$$\sigma_f = \sqrt{\sum \left(\frac{\partial f}{\partial x_k} dx_k\right)^2} \quad \text{(k=1, 2, 3…)} \tag{8}$$

where x_k is the recording parameter and dx_k is the total error of x_k.
We define the objective function (merit function) of GA by:

$$obj = f + \mu\sigma_f \tag{9}$$

Where μ (<1) is a weighting factor that we determine by considering the relative importance of error function σ_f to function f in the design.

It's sure that the more points we select in the grating surface, the more accord with the deviation between the real and expected grooves distribution. But when the selected point number j turns great, the times that calculation cost will be very large, or the efficiency of calculation is too low. We need find a method to improve the efficiency.

Let the selected point number $j \to \infty$, then:

$$\begin{aligned}\lim_{j\to\infty} f &= \lim_{j\to\infty} \frac{1}{j}\sum_{i=1}^{j}[r_1 + r_2 w_i + r_3 w_i^2 + r_4 w_i^3]^2 \\ &= \lim_{j\to\infty} \frac{1}{jdw}\sum_{i=1}^{j}[r_1 + r_2 w_i + r_3 w_i^2 + r_4 w_i^3]^2 dw \\ &= \lim_{j\to\infty} \frac{1}{jdw}\int_{w_{\min}}^{w_{\max}}[r_1 + r_2 w_i + r_3 w_i^2 + r_4 w_i^3]^2 dw\end{aligned} \tag{10}$$

Substitution of Eq. (3) into Eq. (10) yields:

$$\lim_{j\to\infty} f = \frac{1}{(w_{\max} - w_{\min})}\int_{w_{\min}}^{w_{\max}}[r_1 + r_2 w_i + r_3 w_i^2 + r_4 w_i^3]^2 dw \tag{11}$$

Let:

$$w_{\max} = -w_{\min} = w_0 \tag{12}$$

and:

$$\begin{aligned}h &= \frac{1}{(w_{\max} - w_{\min})}\int_{w_{\min}}^{w_{\max}}[r_1 + r_2 w_i + r_3 w_i^2 + r_4 w_i^3]^2 dw \\ &= r_1^2 + \frac{1}{3}w_0^2(2r_1r_3 + r_2^2) + \frac{1}{5}w_0^4(r_3^2 + 2r_2r_4) + \frac{1}{7}w_0^6 r_4^2\end{aligned} \tag{13}$$

Then the objective function can be expressed by:

$$obj^* = h + \mu\sigma_h \tag{14}$$

where:

$$\sigma_h = \sqrt{\sum(\frac{\partial h}{\partial x_k} dx_k)^2} \quad \text{(k=1, 2, 3…)} \tag{15}$$

3. DESIGN EXAMPLE

We now consider a holographic VLS plane grating for a position sensor as a design example to demonstrate the capability of our method. The grating size is $50mm \times 10mm$. The expected groove density is expressed by:

$$n_e(w) = 526.32 \times (1 + 1.4737 \times 10^{-2} w + 2.1717 \times 10^{-4} w^2 + 3.2005 \times 10^{-6} w^3) \tag{16}$$

Here we assume that the recording laser in our example is Kr^+ laser light, the wavelength of it is $\lambda_0 = 4.131 \times 10^{-4} mm$. We optimize the recording parameters by minimizing the objective function in the design.

3.1. Spherical wave fronts recording

First we consider a recording system consisting of two point sources for the sake of simplicity, this system generates two spherical wave fronts.

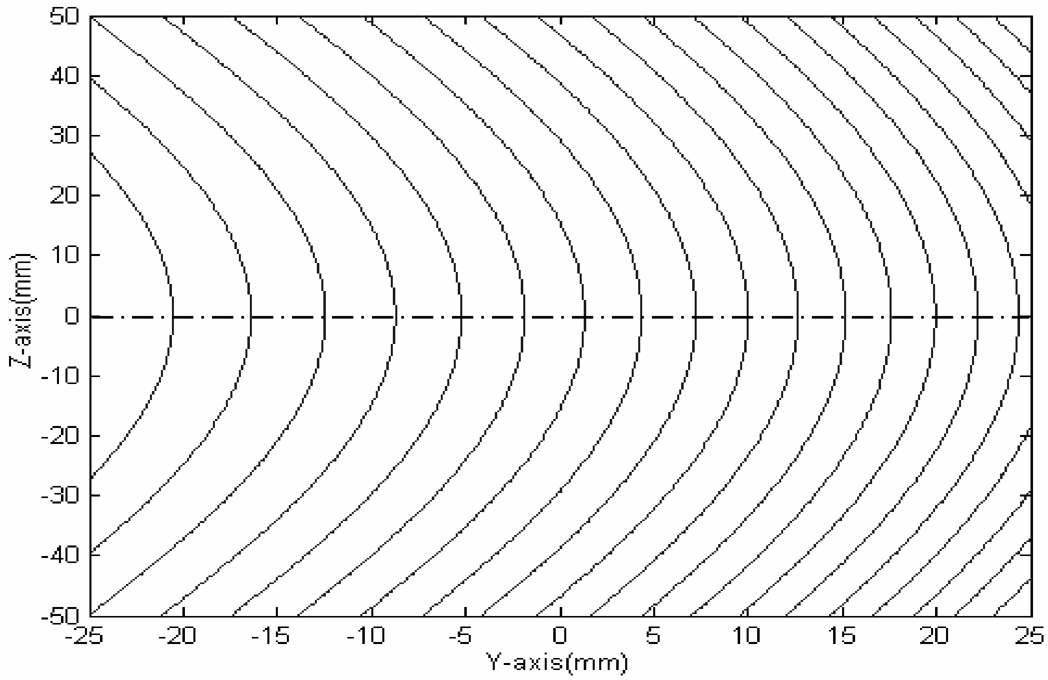

Fig. 3. Schematic of line-profiles of the grating with two spherical wave fronts recording

There are four recording parameters in this system: r_C, γ, r_D and δ. We assume that μ=0.8, $dr_C = dr_D = 1mm$, $d\gamma = d\delta = 0.001 rad$. The calculation result is:

r_C =115.0935mm, γ =0.8744rad, r_D =158.9856mm, δ =1.4108rad, h =43.0674, σ_h =0.0453, obj =43.1037. The line-profiles of the grating with these recording parameters are shown in Fig. 3. The grating size in Fig. 3 is $50mm \times 100mm$ to clearly display the curve tendency of the groove.

The groove density distribution with those recording parameters is shown in Fig. 4.

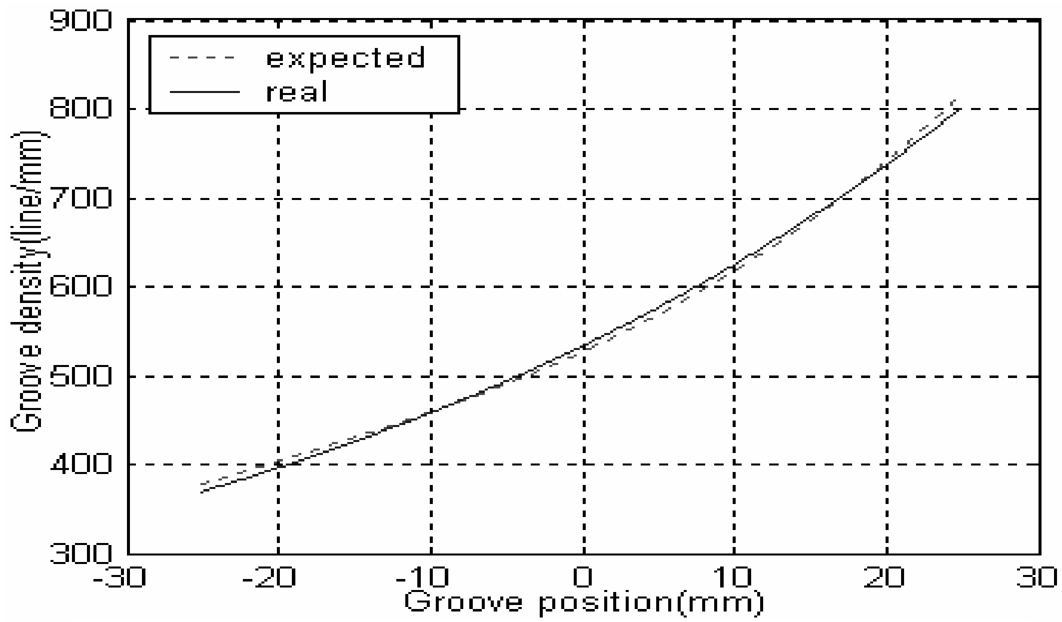

Fig. 4. Schematic of groove density distribution with two spherical wave fronts recording

3.2. **Aspheric and spherical wave fronts recording**

Table 1. Recording parameters of aspheric and spherical wave fronts recording

	$r_C(mm)$	$\gamma(rad)$	$p_D(mm)$	$q_D(mm)$	$\delta(rad)$	$\eta_d(rad)$	h (10^{-15})
No.1	902.7900	0.1505	452.9500	640.9300	0.3762	0.8858	178.2631
No.2	856.4731	-1.4777	863.8348	936.7188	-0.8919	0.3111	0.09624

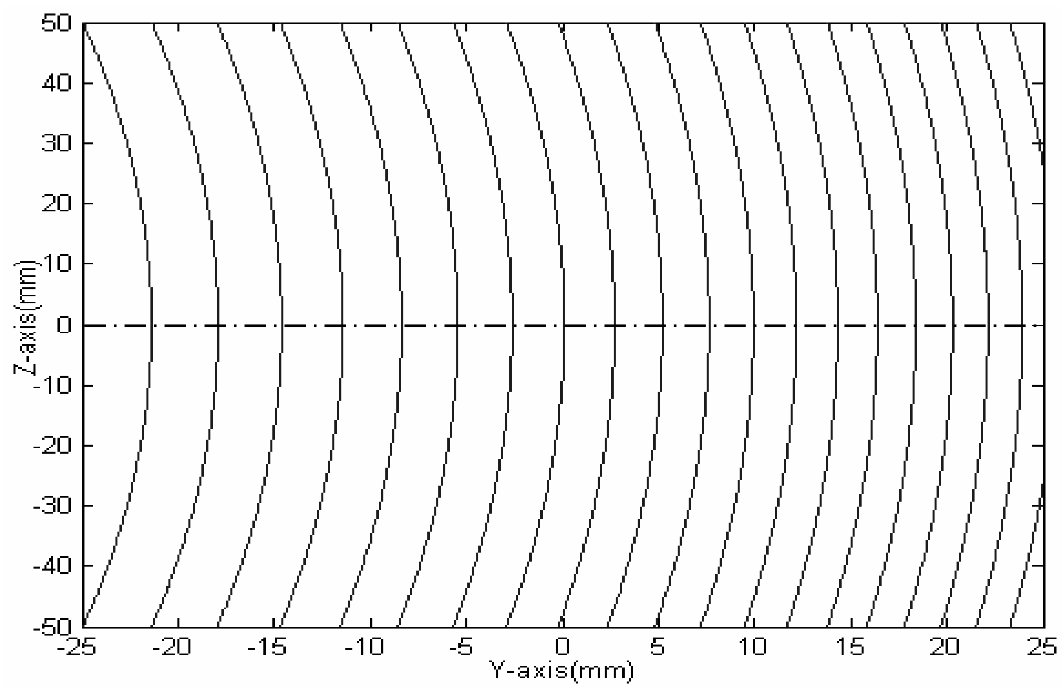

Fig.5. Schematic of line-profiles of the grating with parameters No.1

Second we consider a recording system consisting of a spherical mirror, M_2, and two point sources. This system generates an aspheric and a spherical wave front. Here we assume that the radius of M_2 is $1000mm$.

There are six recording parameters in this system: $r_C(r_C = p_C + q_C), \gamma, p_D, q_D, \delta, \eta_D$. On this condition σ_h is very complex, we omit it and calculate h only in the objective function for the sake of simplicity. We use GA and local search to determine the design parameters. The results are summarized in Table1.

The line-profiles of the grating with parameters No.1 and No.2 are shown in Fig. 5 and Fig. 6, respectively. For the position sensor we hope that the line-profiles are straight, we select No.1 parameters as recording parameters.

The groove density distribution with parameters No.1 is shown in Fig. 7. From Fig. 7 we know that the real distribution quite accord with the expected distribution.

The correspondent groove parameters of recording parameters which are shown in 3.1 and No.1 are listed in Table 2. From Table 2 we know that we can obtain the correct coefficients by introducing an aspheric wave front into the system. From Fig. 3, Fig. 5 and Fig. 6 we also can see that the line-profiles of holographic VLS plane gratings with aspheric and spherical wave fronts recording are not usually straighter than that with spherical wave fronts recording.

And as a test of calculation efficiency, we optimize the recording parameters with the objective function h and f with $j = 1001$, respectively. The former efficiency is 3~4 times better than the later.

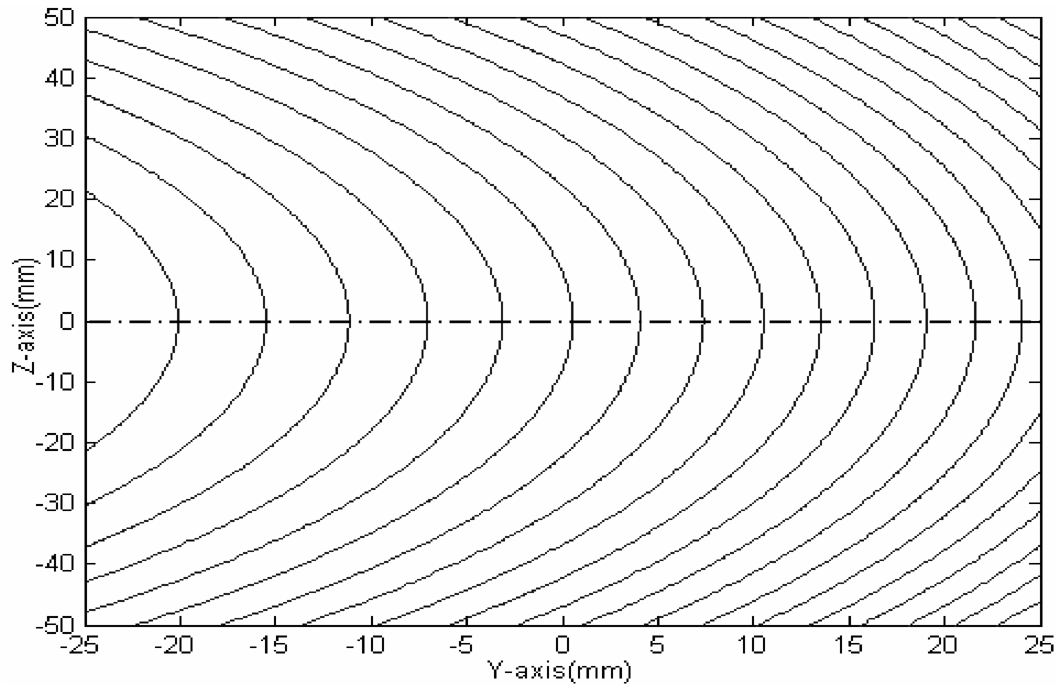

Fig. 6. Schematic of line-profiles of the grating with parameters No.2

4. CONCLUSIONS

A method of recording parameters optimization of holographic VLS gratings has been developed by introducing the genetic algorithm into the optimization program. We consider the effect of the recording parameters errors in the objective function of genetic algorithm to obtain a steady set of recording parameters. The validity and the design capability of the present method are demonstrated in the design example of a VLS plane grating for a position sensor.

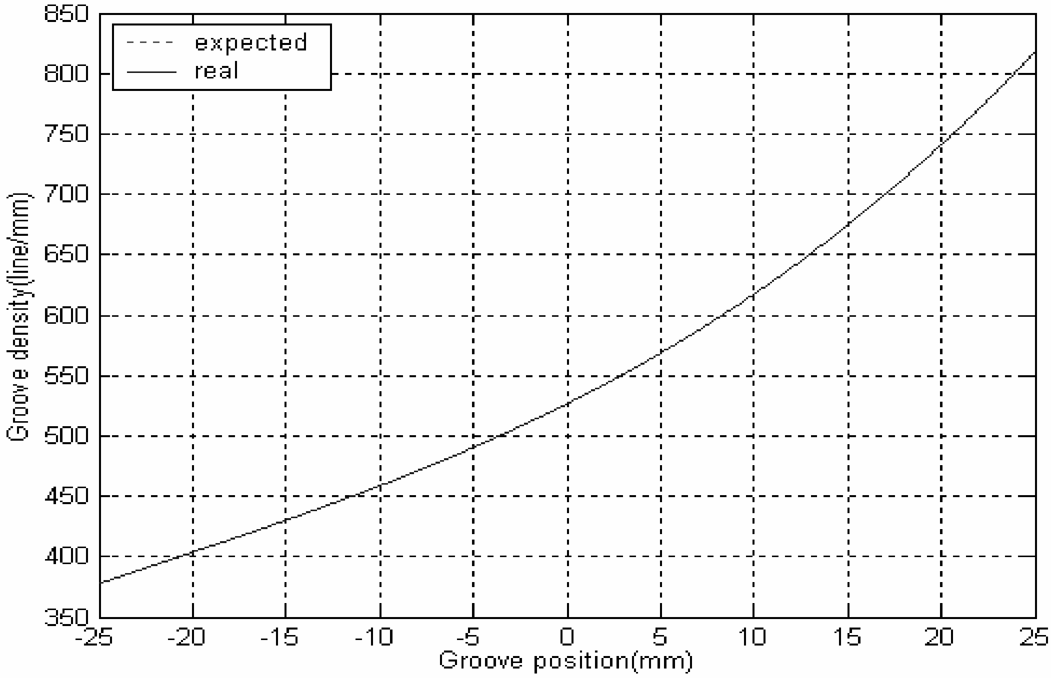

Fig. 7. Schematic of groove density distribution with parameters No.1

Table 2. Holographic VLS grating groove parameters

Groove parameters	$n_0\ (l/mm)$	$b_2\ (10^{-2}/mm)$	$b_3\ (10^{-4}/mm^2)$	$b_4\ (10^{-6}/mm^3)$
Expected parameters	526.3200	1.4737	2.1718	3.2006
Spherical wave fronts recording	532.6683	1.5520	1.5568	1.1358
Aspheric and spherical wave fronts recording	526.3200	1.4737	2.1718	3.2006

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