Design of DOE for beam shaping with highly NA focused cylindrical vector beam

Yiqiong Zhao¹, Qiwen Zhan², Yong-Ping Li¹*

¹. Physics Department, University of Science and Technology of China
². Electro-Optics Graduate Program, University of Dayton

ABSTRACT

Focus shaping with highly focused cylindrical vector beam is an interesting and important topic in both applied optics and physical optics. In this paper, we describe optimization algorithms to design diffractive optical elements for various beam shaping applications. “Optical bubbles” with desired size and numbers for trapping particles and flat-top focusing with low side-lobe to improved micro printing are proposed. Ultra small focus spot with long depth of focus without energy split to extend the region around the image plane is also presented.

Keywords: diffractive optical element, beam shaping, cylindrical vector beam, optical trapping, flat-top, depth of focus

1. INTRODUCTION

The three dimension electromagnetic field distribution near focus of high numerical aperture (NA) has always drawn considerable attention for many applications in data storage, lithography, as well as particle trapping. The key technique to realize these different kinds of functions is how to modulate the incoming light and design the optical elements. As the unique properties of light beam, the polarization plays an important role and provides more freedom in the optical system design. In recent years, a novel vector beams with cylindrical symmetry both in field amplitude and polarization are incorporated into highly focused system, and proved to expand the functionality and enhance the capability of optical system, such as narrow the focus spot into an ultra-small size with a long depth of focus [1], generate flattop focus for a general cylindrical vector beam [2], build an optical “bubble” tweezers [3], and et al. On the other hand, optical elements, especially diffractive optical elements (DOE), offer us completely controllable freedom for beam shaping. As we known DOE influence the wave-front by diffraction and can convert laser beams to nearly arbitrary distribution to form desired distribution by optimum algorithms.

The design of DOE for beam shaping is well known a typical inverse optimization without analytical expression. Various practical algorithms have been developed and applied for resolving this problem. They are divided into two major kinds. One kind is iteration algorithms, such as GS (Gerchberg-Saxton) [4] and IO (Input-Output) [5,6] algorithm and another kind is optimization algorithms, such as SA (Simulated Annealing) [7,8] and GA(Genetic algorithm) [9,10]. Hybrid algorithms of methods above have been proved more efficiently in recent years [11,12].

In this paper, optimization algorithms are presented to design the DOE for a variety of beam shaping applications. With

* Corresponding author, Email: liyp@ustc.edu.cn
appropriate definitions of cost function, we can obtain different focusing field strength distribution to meet the requirements for different applications. The simulation results show that field strength can be continuously adjusted and near-optimum profile can be obtained with the optimal DOE designs. The “optical bubbles” which form different stable three dimension wells can be obtained and may find applications in optical trapping, and biomedical imaging. Flattop focusing with low side-lobe within small volume can be obtained with optimally designed DOE for printing, micro-fabrication and photolithography. A controllable ultra small focus with extended depth of focus without energy split in the axial direction can also be achieved with this focus shaping technique, which may find applications in high-density optical data storage and micro-fabrication.

2. THEORY

Consider our focusing system is setup with a DOE in front of high NA lens with an aperture, as shown in figure 1. The focal plane is placed many wavelength away from the geometric focus. The incoming light is general cylindrical vector beam (figure 2), which can be written as

\[ \vec{E}(r, \phi, 0) = A \begin{pmatrix} \cos \phi_0 e_r \\ \sin \phi_0 e_\phi \\ 0 \cdot e_z \end{pmatrix}, \]

where \( e_r, e_\phi, \) and \( e_z \) are the unit vectors in the radial, azimuthal, and z direction, respectively, \( A \) is a constant factor, and \( \phi_0 \) is the rotated angle of the polarizations from the radial direction.

![Diagram of focusing system with DOE and aplanatic lens](image-url)
Various methods have been studied to generate or pick out these beams for about several decades. These methods are including interferometric or liquid-crystal polarization converters [13], computer generated holograms [14], optical fiber selection [15] and et al. The electronic field near focus can be calculated using the Debye approximation and expressed as:

$$E(r, \phi, z) = A \int_{\theta_{\min}}^{\theta_{\max}} \cos^2(\theta) T(\theta) e^{i 2z \cos(\theta)} \cdot \begin{pmatrix}
\cos \phi_0 \sin \theta \cos \partial J_{\lambda}(kr \sin \theta) \\
\sin \phi_0 \sin \theta \partial J_{\lambda}(kr \sin \theta) \\
i \cos \phi_0 \sin^2 \partial J_{\lambda}(kr \sin \theta)
\end{pmatrix} d\theta,$$

(2)

where $\theta_{\max}$ is the maximal angle determined by the numerical aperture of the object lens, $\theta_{\min}$ is the minimal angle of annular incident beam, $\cos^2(\theta)$ is the apodization factor for an aplanatic lens, $k$ is the wave and $J_n$ is the Bessel function of the first kind with order $n$. $T(\theta)$ is the transmission function of DOE, the length unit is normalized to wavelength, therefore, $\lambda = 1$.

DOE design process is a phase retrieval problem. As the main features of these problems, which project to our system, the electronic field distribution of incident beam and desired profile near focus are known, while the key is searching for a specific distribution of DOE to meet equation (2) as well as possible. In practice, the different methods are generally combined and hybrid optimization algorithms are used to enhance the quality of solutions and improve the performance. Here we used global search algorithm with cost function as criterion. Cost function is defined as

$$\text{Cost} = f(M_{\text{real}}, M_{\text{ideal}}),$$

(3)

where $M_{\text{real}}$ is the real-time model which depend on the propagation of focus system with real-time distribution of DOE, $M_{\text{ideal}}$ is the desired ideal model built up according to different demands. $f$ is the function of evaluation criterion.

Fig. 2 Generalized cylindrical vector beam with $\phi_0$ rotation from the purely radially polarization.
In view of the feasibility of fabrication, we simplified the transmission function of DOE as circular symmetry step type profile with expression as

$$T(\theta) = \begin{cases} T_1 & \text{if } \theta_{\min} \leq \theta < \theta_1 \\ T_2 & \text{if } \theta_1 \leq \theta < \theta_2 \\ \vdots & \vdots \\ T_N & \text{if } \theta_{N-1} \leq \theta < \theta_{\text{max}} \end{cases}$$

(4)

$T_i$ is chosen as 1, 0, or -1, where $N = 10$.

$$\theta_i = \begin{cases} \theta_{\min} & i = 0 \\ \arcsin\left(\frac{NA - \sin(\theta_{\min})}{N} \cdot i + \sin(\theta_{\min})\right) & i \neq 0 \end{cases}$$

(5)

In the following section, we present several new applications for cylindrical vector beam shaping that are resulted from the design of DOE for highly focused system.

3. DESIGN OF DOE FOR CYLINDRICAL VECTOR BEAM SHAPING

3.1 Optical bubbles

Optical trapping, also called “optical tweezers”, which is first demonstration by Ashkin [16,17], is a powerful tool to trap particles in three dimension through the balance of gradient force and scattering/absorption force of highly focused laser beam. Based on this non-destructive unique, this technique has many applications in numerous areas, including biology [3], chemistry [4] and physics [5]. Standard tweezers trap particles of high refractive index with bright spot, while inverted tweezers trap particles of low refractive index. Three-dimension optical bubble [3] generated by combination of DOE and highly focused cylindrical vector beam is a novel efficient inverted tweezers. We take this technique a step further by forming and controlling the properties and numbers of bubbles by modulate the wave-front of incident beam with DOE.

To set up desired model for inverted trapping, the field strength near focus should be assumed with “bubble” properties, which means field strength around focus should be higher than that in center. For projecting this character into the cost function, bubble size and depth should be defined to describe this real-time distribution. The difference between the maximum and geometrical focus’ field strength along the optical axis (radial axis) is expressed as the depth of the bubble $D_z (D_r)$, and the full width between the peaks of field strength along the optical axis (radial axis) as the bubble size $S_z (S_r)$. Then the objective cost function is introduced as following

$$\text{Cost} = \left[\alpha \left(\frac{aD_z}{bD_r} + \frac{bD_z}{cS_z + dS_r}\right) + \beta (cS_z + dS_r)\right],$$

(6)

where $\alpha, \beta$ are weighting factor in order to balance the depth and size of the bubble, and $a, b, c, d$ are used to control the proportion of transverse and longitude values.

When trapping individual particle, single bubble with size to fit the particle inside and depth to form stable trap are needed. By locking single bubble distribution and changing the weigh of size and depth to balance each other, different bubbles are achieved in left column of figure 3. Things all obey the physical rules though we try to manipulate it at our
Fig. 3 Optical bubbles formed by illuminating of radially polarized incident beam through an optimum DOE with NAs. The field strength $|E|^2$ appears different kinds of distribution resulted from the definition of ideal models: the bubbles in left column are achieved by locking single bubble and changing the size and depth; and that in right column are from searching the smallest bubbles with desired numbers.
pleasure. The simulation result of left column in figure 3 shows that the bubble size is in inversely proportional to the depth\[^{18}\]. Bubble size can be controlled as small as possible with the cost of deceased depth until it degrades into flat top distribution owing to the overlap effect of the field strength of different field components around the focus. Whereas when the bubble is large enough to avoid this effect, a totally three dimension hollow bubble with zero field strength around the geometrical focus can be obtained.

While to extend the application, creating trapped structure in multiple sites (array) is investigated for irregular particles or more than one particle manipulate. Here we also produce a periodic bubbles chain with trapping every particle in three dimensions in different plane. As shown in the right column of figure 3, hollow bubbles chains are created from searching the smallest bubbles with desired numbers of 1, 3, 5. This technique is based on the coherent beams superimpose their amplitudes with one another in interference and then combine with the diffractive effect to produce special distribution and provides a multi-functional tool for many-potential applications, such as quantum gate, optical lattice, or unraveling DNA condensation.

### 3.2 Flat-top beam shaping

Beam shaping with uniform illumination (flat-top) is applied in many areas, such as microlithography and semiconductor laser Insert and et al. In theory, a complex DOE can make the top as flat as possible with no side lobe, though difficult to realize by experiment. Limited by the technology of fabrication, simple distribution of DOE with high quality of output flat-top shaping is required. Whereas when we chase ideal flat-top field strength, the integral side lobe will be increasing. More flat the top is, the more increase will go with. So besides the flat-top, try to restrain the high side lobe is the other important errand of optimize.

Here we also define the DOE with the initial parameters of equation (4) and (5). Based on the properties of highly focused cylindrical vector beam, the field strength near focus is taken the super-Gaussian forms:

\[
|E_{\text{ideal}}(r,z)|^2 = \exp\left(-\frac{r^{2N_r}}{R^2} - \frac{z^{2N_z}}{Z^2}\right),
\]

where \( R = 1.5\lambda, Z = 4.5\lambda \), the slopes \( N_r = N_z = 3 \). The cost function is then defined as

\[
\text{Cost} = g\left(|E_{\text{real-time}}|^2 - |E_{\text{ideal}}|^2\right),
\]

where \( g \) function is to balance the optimization of different emphasis, for example \( r \) direction or \( z \) direction, side lobe or main lobe. The simulated distribution with flat-top and slow side lobe in the \( r \) direction around the focus when \( \phi_0 = 0^\circ \) is obtained as shown in Fig.3. Increasing \( \phi_0 \) properly will improve uniform illumination by inducing the donut field strength from azimuthally polarization\(^{2}\). The optimum results show that the flat-top degrees of \( r \) direction and \( z \) direction are restrict with each other: longer depth of focus (flat-top in the \( z \) direction) will cause large side lobe in the \( r \) direction; while flat-top in the \( r \) direction will reduce the edge abruptness in the \( z \) direction. More complex distribution of DOE will improve this disadvantage due to more freedom for beam shaping.
To match the exponential growth requirement for high-density optical storage and resolution, focusing spots beyond the diffraction limit with long depth of focus (DOF) is always very important for optical engineers and scientists. In recent years, modulate the polarization of incoming light to narrow the focus spot is of increasing interest. In this area, cylindrical vector beams are special solutions of Maxwell’s equations to improve the resolution. These beams can form highly confined field strength near the focus along axis direction with long DOF. Especially, the spot of longitude component field strength of highly focused radial polarization beam is smaller than that of linearly polarization\(^\text{[19]}\). With introducing a binary phase plate, Ching-Cherng Sun and Chin-Ku Liu\(^\text{[1]}\) presented a design for focusing an incoming light into an ultra small spot with a long DOF. But their results showed that the greatest energy along the optical axis is located near twice the wavelength in front of and behind the focus point. This phenomenon will introduce disadvantage and limit applications based on the split of energy distribution. We make their technique a little further with optimum design to form flat-top distribution in the \(r\) direction by illuminating of radially polarized incident beam through an optimum

\[ \text{(a) Total field strength} \]

\[ \text{(b) Field strength at the focal plane} \]

\[ \text{(c) Field strength at the axis plane} \]

3.3 Ultra small focus with extended depth of focus

To match the exponential growth requirement for high-density optical storage and resolution, focusing spots beyond the diffraction limit with long depth of focus (DOF) is always very important for optical engineers and scientists. In recent years, modulate the polarization of incoming light to narrow the focus spot is of increasing interest. In this area, cylindrical vector beams are special solutions of Maxwell’s equations to improve the resolution. These beams can form highly confined field strength near the focus along axis direction with long DOF. Especially, the spot of longitude component field strength of highly focused radial polarization beam is smaller than that of linearly polarization\(^\text{[19]}\). With introducing a binary phase plate, Ching-Cherng Sun and Chin-Ku Liu\(^\text{[1]}\) presented a design for focusing an incoming light into an ultra small spot with a long DOF. But their results showed that the greatest energy along the optical axis is located near twice the wavelength in front of and behind the focus point. This phenomenon will introduce disadvantage and limit applications based on the split of energy distribution. We make their technique a little further with optimum design to form
a controllable ultra small focus with extended depth of focus without energy split in the axial direction. Actually, the problem is to search for the distribution near focus with long DOF (flat-top in the \( z \) direction), small spot size and greatest energy at focus plane. So we should only put a little qualification in the cost function of equation (8). The optimum results is shown in figure 5 that the radius of the main lobe in the focus plane is 27% smaller than the diffraction-limited one with longer DOF even than the result presented in reference [1]. The flat-top in the \( z \) direction is achieved without visible split. Long depth of focus with small spot radius is especially important if the test point isn’t fixed very well and can extend the region around the image plane with sharp record.

![Graphs and images showing field strength components](image)

(a) Longitude field strength component

(b) Field strength at the focal plane

(c) Field strength at the axis plane

Fig.5 Longitude field strength component with small spot size and long DOF of highly focused radially polarized incident beam through an optimum DOE.
4. CONCLUSION

In summary, we have described beam shaping of cylindrical vector incident beam with high NA focus system. Using DOE designed by optimum algorithms, the field strength near focus is controlled to meet requirements. Examples are proposed for different applications, such as creating 3D bubbles for optical trapping, smoothing flat-top focus for laser printing and material processing, and ultra-small spot size with long DOF for microscopy.

REFFERENCE
