Computer generated hologram of asymmetry fractional Fourier transform

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ABSTRACT

A new optical encryption technique based on computer-generated hologram (CGH) and fractional Fourier transform (FRT) is presented. And the algorithm of making asymmetry FRT CGH is provided in this paper. In this method, the fractional Fourier transform of the input image is performed by two one-dimensional FRT with different orders in the *x* and *y* directions in cascade. With Lohmann Ⅲ detour phase encoding method and computer plotting program, the transformed image is encoded and fabricated into CGH on the computer. Then a piece of asymmetry fractional Fourier transform CGH (AFRTCGH) of original input image is obtained. In order to reconstruct the encoded image, a special fractional Fourier transform systems with two special cylinder lens' are needed. Namely, only when the transform order in each direction is respectively matched with that of the asymmetry fractional Fourier transform CGH, can the encoded image be reconstructed exactly. Because of its particularity of image reconstruction, it is regarded as a new optical security system and can be used in anti-counterfeiting. When it is used to encrypt image or to anti-Counterfeit, anti-counterfeiting intensity can be improved greatly. So it has very high applying value.

Key words: fractional Fourier transform (FRT), computer generated hologram (CGH), detour phase coding method, optical security, asymmetry, anti-counterfeit, optical encryption, transform order, image reconstruction, simulation.

1. INTRODUCTION

Computer generated hologram (CGH) is based on diffractive optical elements, communication modulated theory and digital computer, which is a kind of digital holograph fabricated by the aid of computer. .It has several unique advantages over optical hologram, including the following: low noise, easy replication, flexible function, wide applicable range. So the computer generated hologram has been widely used in different domains such as image display, optical correlation recognition, beam shaping, interferometry, laser scanning[1] etc. In 1980 Namias has introduced mathematically fractional Fourier transform to the solution of differential equation in quantum mechanics and MacBride given its more strict definition[2-7]. It then has been applied to optical domains when Mendlovic and Ozaktas put forward the realization of fractional Fourier transform performed by square graded- index optical fibers in 1993. Later Lohmann etc. have successfully realized the fractional Fourier transform by discrete lens system. The fraction order of the fractional Fourier transform as a new degree of freedom has been greatly enriched the content of the optical communication processing and has practical application in many aspects, for instance: fraction convolution, fraction correlation and filtering in the fractional Fourier transformation field. At the same time it has also been a potent tool in the analysis of light beam transmission and optical system.

In this paper a new optical encryption technique based on computer generated hologram of asymmetry fractional Fourier transform is presented. In this method, the fractional Fourier transform of the input image is performed by two one-dimensional FRT of different orders in the x and y directions in cascade. To smooth the FRT domain spectrum, a random phase exponential function is necessarily introduced into the input image before the transform. With Lohmann Ⅲ detour phase encoding method and computer plotting program, the transformed image is encoded and fabricated into CGH on the computer. Then through computer output, laser print and developer, an asymmetry fractional Fourier transform CGH of original input image is obtained. In order to reconstruct the encoded image, a special fractional Fourier transform systems with two special cylinder lens' are needed. Namely, only when the transform order in each direction is respectively matched with that of the asymmetry fractional Fourier transform CGH, can the encoded image be reconstructed exactly. On the other hand, the encrypted hologram can be made conveniently and flexibly by using computer-generated hologram technique, so when it is used to encrypt image or to anti-Counterfeit, not only anti-counterfeiting intensity is improved greatly, but operation and application are more convenient. So it has very high applying value. At the end of this paper, the computer simulation results are given.

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2. ALGORITHM ANALYSIS

The definition of the fractional Fourier transform has many modes, the following is the fractional Fourier transform mathematical definition mode proposed by Namias in 1980. If the input function is *f*(*x*), the fractional Fourier transform of order α is defined as:

$$F^{\alpha}\{v\} = \frac{exp(i\frac{\pi}{4} + i\frac{\phi}{2})}{[|sin(\phi)|]^{1/2}} \int_{-\infty}^{\infty} f(x) exp[\frac{i\pi(x^2+v^2)}{tan\,\phi} - \frac{i2\pi xv}{sin\,\phi}]dx \tag{1}$$

$where\ \phi = \alpha\pi/2, \alpha$ is the real fractional order.

To discuss conveniently, the equation (1) can be rewritten to:

$$F^{\alpha}\{f\} = c\,exp[\frac{i\pi v^2}{tan\,\phi}] \int_{-\infty}^{\infty} \{f(x) exp[\frac{i\pi x^2}{tan\,\phi}]\} exp[-\frac{i2\pi xv}{sin\,\phi}]\,dx \tag{2}$$

$where\ \phi = \alpha\pi/2$

The aim of such recomposition is to emphasize a chirp term $\exp(\frac{i\pi x^2}{\tan\phi})$)and a Fourier transform term $exp(-\frac{i2\pi xv}{sin\,\phi})$ with a scale factor. In order to calculate the expression, the procedure is as followings:

Step (1): Dispersing the input and chirp signal respectively, thus we obtain the discrete function $f(n), (-N/2 \le n \le N/2)$ and the discrete chirp signal $\exp(\frac{i\pi n^2}{\tan\phi})$, where N is the total number of samples..

Step (2): We multiply the function $f(n)$ by the corresponding chirp term $exp(\frac{i\pi n^2}{tan\,\phi})$ to obtain the complex signal.

Step (3): Taking the Fourier transform of the complex signal in step (3).

Step (4): Taking the Scale transform of the result of step (3), the transform factor is $1/sin\,\phi$.

Step (5): Multiplying the constant and the chirp factor $exp(\frac{i\pi v^2}{tan\,\phi})$.

So we can perform the fractional Fourier transform by transferring the fast Fourier transform algorithm and greatly decrease the number of computation. It provides a powerful tool for us in designing the computer generated hologram of fractional Fourier transform.

3. THE COMPUTER GENETATED HOLOGRAM OF AFRTCGH

Assuming that the input image function is *f* (*x*, *y*), we can obtain F1 by one-dimension discrete fractional Fourier transform of order α of the column vector of *f*(*x* , *y*) by the use of the above FRT transform algorithm in section 2. And we obtain F2 by performing the FRT transform of order β of the row vector of F1. Then we take the transpose of F2 to obtain F3 which is the desired result of asymmetry fractional Fourier transform. F3 is encoded by the Lohmann Ⅲ detour phase coding method to produce the asymmetry fractional Fourier transform generated hologram on the screen of

the computer. Then through computer output, laser printing, developer and optical microfilm, a desired piece of AFRTCGH is obtained.

4. OPTICAL RECONSTRUCTION

Here we place the AFRTCGH in the input plane, as shown in Fig1, the inverse fraction Fourier transform of order α in the x direction can be realized through twice phase transforms and one-time Fresnel diffraction. And that of y direction can be realized by twice Fresnel diffractions and one-time phase transforms. The focal length of

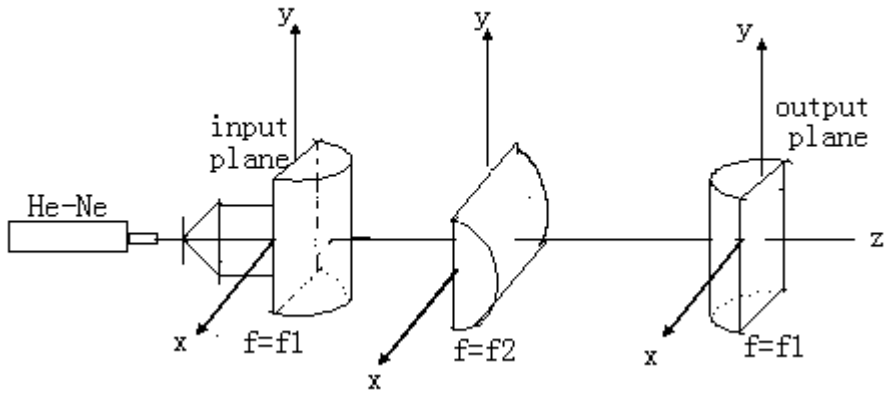

Fig.1. Schematic of optical reconstruction device

cylinder lens in Fig.1 is f_1, f_2 respectively. According to the conditions of the optical implementations of fractional Fourier transform:

$d_1=f_1(1-\cos\phi_1)$; where $\phi_1=\alpha\pi/2$, d_1 is the distance between the input place and output place;

$d_2=f_2(1-\cos\phi_2)$; where $\phi_2=\beta\pi/2$, d_2 is the distance between the input place and output place;

If $d_1=d_2=d$, the equation exists

$$f_1(1-\cos\phi_1)=f_2(1-\cos\phi_2) \tag{3}$$

When f_1 and f_2 meet the following condition:

$$f_1=f_2\left(\frac{1-\cos\frac{\beta\pi}{2}}{1-\cos\frac{\alpha\pi}{2}}\right) \tag{4}$$

the inverse fraction Fourier transform of different orders in the *x* and *y* directions can be realized in cascade simultaneously. So the optical reconstruction is completed.

5. COMPUTER SIMULATION RESULTS

To investigate the performance of asymmetry fractional Fourier transform CGH, we adopt a 64×64 pixel letter e as the input image information, as shown in Fig.2(a).Then the asymmetry fractional Fourier transform of order $\alpha=0.75$ along x direction and $\beta=0.75$ along y direction of letter e is performed respectively. To smooth the fraction domain spectrum of the input image, a random phase exponential function is introduced into the input image before the asymmetry fractional Fourier transform is taken. With Lohmann Ⅲ detour phase encoding method and computer plotting program, the transformed image is encoded and fabricated into CGH on the computer. Then through computer output, laser print and developer, an asymmetry fractional Fourier transform CGH of letter e is obtained, as shown in Fig.2 (b). Fig2 (b) shows only a part of the enlarged computer generated hologram. To receive the desired AFRTCGH,

the enlarged CGH must be microfilmed on the film. In our experiment, the size of original CGH is 300mm×300mm and the minification is 80. Then place the AFRTCGH of

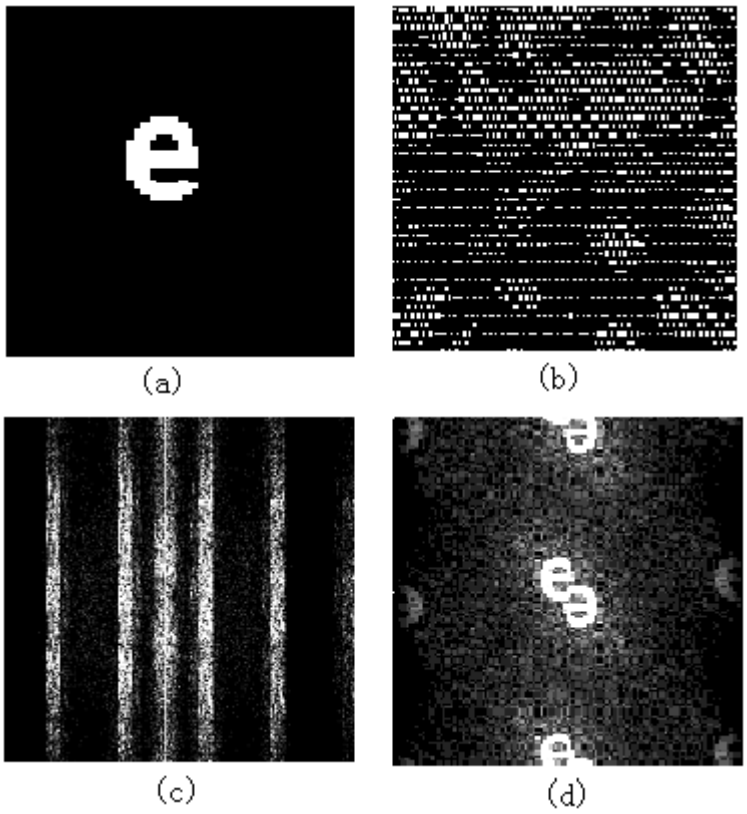

Fig.2.Simulated results by computer
(a) original input object; (b) asymmetry CGH of original object; (c) reconstructed result by order $\alpha=0.9$, $\beta=1$; (d) reconstructed result by $\alpha=0.9$, $\beta=0.75$

letter e in the input scene of the optical reconstruction system, as shown in Fig.1. We obtained the simulation reconstruction results of AFRTCGH of order of $\alpha=0.9$, $\beta=1$ and $\alpha=0.75$, $\beta=0.75$ respectively on the computer. The reconstruction results are shown in Fig.2(c)-(d). The simulation results indicate that only when the reconstruction order is $\alpha=0.75$, $\beta=0.75$, namely the reconstruction order corresponds to that in the recorded system, the original input image information can be reconstructed.

6. Phase encryption using asymmetry FRT

Ordinarily the images to be encrypted are intensity representation. Any intensity detector can detect the amplitude image. If the original image is phase encoded and then encrypted for further uses, it is impossible to acquire the information content of the phase image, unless we use some technique for converting the phase image into an amplitude image. Therefore, fully phase encryption is more secure than amplitude encryption. Neto has proposed an image-encoding scheme to encrypt images in phase masks using the phase-contrast technique and a random phase distribution. Towghi *et al* have discussed a fully phase image encryption technique that uses the double random-phase encoding method. They have shown that fully phase encryption performs better than amplitude-based encryption in the presence of additive noise with respect to the mean square error.. Tan *et al.* have reported a secure optical storage that uses a fully phase encryption. They have shown that a fully phase-based encryption system generally performs better than an amplitude-based encryption system when the system bandwidth is limited by a moderate amount. The problem with the fully phase encryption arise only during decryption. Optical decryption can be implement in the common path interferometer by use of single phase or if desired a combined phase key. If we introduce two random phases into the

asymmetry computer generated hologram, the anti-counterfeiting intensity can be improved more greatly.

7. CONCLUSIONS

We have identified an optical encryption technique by the use of fractional Fourier transform. With this technique, the coding and decoding of the input images can be realized conveniently and simply. We obtained the desired experimental results by computer simulation. From the above analysis, it can be seen that only when the order of fractional Fourier transform in the reconstruction system matched to that in the recorded system, the original objects can be reconstructed. Because of this particularity of image reconstruction, it is regarded as a new optical security system and can be used in anti-counterfeiting. When it is used to encrypt image or to anti-Counterfeit, anti-counterfeiting intensity can be improved greatly. So it has very high applying value.

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