A robust blind deconvolution based on estimation of point spread function parameters

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ABSTRACT
At present, in the field of image processing, the main algorithm to restore the blurred image is the blind deconvolution. But most of the blind deconvolution methods have to iterate a large amount of times and the result is also unsatisfactory. In this paper, a new blind deconvolution algorithm is proposed, which, consisting of two steps, is based on simultaneous estimating the specimen function and the parameters of the point-spread function (PSF). Firstly, it uses the expectation maximization algorithm (EM) to iterate the specimen function; secondly it uses the conjugate gradient method to estimate the parameters of the PSF. The mathematical model ensures that all the constraints of the PSF are satisfied, and the maximum-likelihood approach ensures that the specimen is nonnegative. In this paper, the general Gauss function is used to be as the PSF. In the experiment, it can successfully restore both the two-dimensional and three-dimensional images within limited times of iteration.

Keyword: blind deconvolution, point spread function parameters, maximum likelihood estimation.

1. INTRODUCTION
In some fields of biology, three-dimensional fluorescence microscopy is usually used to observe the blurred biological specimen\(^1\). When the microscope is set to focus on the different planes of the specimen, it will produce a series of optical slices. These two-dimensional optical slices will have some different degrees of the blur produced by the light from the in-focus and out-of-focus planes. The main cause of the degradation is the diffraction of the microscope’s objective. The point-spread function (PSF) is characterized by the degradation\(^2\). In the experiment, the point-like object that is used to record the PSF was so small that the image is rather dim. So the PSF recorded in this way is unidealistical, some will even be obscured by the noise. Some necessary parameters that are used to compute the PSF are usually unknown, so in many situations, the theoretical approach can’t get the good result. At present, most people use the blind deconvolution approach to recover the images that ask them to simultaneously estimate the sample function and the PSF\(^3\)-\(^5\). Usually this approach will have non-unique solutions, so in order to get the unique solution they have to add some constraints, such as non-negative, band limitation on it.

In this article, we use the parametrical blind deconvolution approach to recover the blurred image. We suppose that the PSF follows a mathematical model, which was determined by a series of parameters. Because of the mathematical model being used, the PSF will automatically satisfy all the constraints that are added by the model. As above-mentioned, this algorithm aims at simultaneously estimating the specimen function and the parameters of the PSF.
2. PARAMETRICAL BLIND DECONVOLUTION

From the aspect of the statistical photons, the image progress recorded by the microscope is as follows: the photons of the light source reached the specimen randomly; the photons are absorbed by the specimen randomly; the absorbed photons are excited randomly; the excited photons (represented by \( s(x_o) \)) partly entered into the microscope randomly; the photons which can access the microscope are partly able to reach the image plane randomly; the photons (represented by \( g(x_i) \)) which can reach the image plane can partly be detected randomly. The image recorded by the optical microscope conforms for the Poisson distribution\(^6\), and the mean value is \( u(x_o) = h(x_o) \otimes s(x_o) \). In this article the PSF is supposed to depend on a series of parameters: \( \Theta = \{ \theta_1, \theta_2, \theta_3, \cdots \} \).

From the image model, there is:
\[
g(x_i) = h(x_o) \otimes s(x_o) + \eta(x_o)
\]  
\( (1) \)

From the Poisson distribution, there is\(^8\):
\[
P(g(x_i) \mid s(x_o)) = \prod_n \frac{u(x_o)^{g(x_i)} e^{-u(x_o)}}{g(x_i)!}
\]  
\( (2) \)

So the log-likelihood function about the \( s(x_o) \) is:
\[
\ln P(g(x_i) \mid s(x_o)) = \sum_n [g(x_i) \ln u(x_o) - u(x_o) - \ln \{g(x_i)\}]
\]  
\( (3) \)

In order to be convenient for the computation and representation, the integrated form of the log-likelihood function is\(^7\):
\[
L[\ln P(g(x_i) \mid s(x_o))] = - \int \left[ h(x_i - x_o) s(x_o) dx_o - g(x_i) \ln \left\{ h(x_i - x_o) s(x_o) dx_o \right\} \right] dx_i
\]  
\( (4) \)

Where, in order to simplify the computation, the PSF is supposed to basically conform to the Gauss model. In this model, we set the variance \( \sigma \) of the Gauss function to be the PSF’s parameters. So the form of point-spread function is:
\[
h(\sigma) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{X^2}{2\sigma^2}\right]
\]  
\( (5) \)

In the maximum likelihood algorithm, in order to get the maximum likelihood function, we use EM algorithm\(^9\). During the iteration, when updating the specimen function, the PSF is set to be constant (equals to set its parameters be constant); when updating the PSF (equals to update its parameters), the specimen function is set to be constant.

In the ML algorithm, in order to get the extreme value of the likelihood function, from the equation (4) the derivation of the log-likelihood function is set to be:
\[
\frac{\partial L}{\partial s} = - \left[ \int \frac{h(-x_o)dx_o}{h(x_o) \otimes s(x_o)} \otimes h(-x_o) \right] dx_i = 0
\]  

so we can get:

\[
\frac{g(x_i)}{h(x_o) \otimes s(x_o)} \otimes h(-x_o) = H_o
\]  

From the iterative formula, it can deduce:

\[
s(x)^{(t+1)} = \frac{s}{H_o} \ast \left[ \frac{\otimes g(x)}{g(x)} \right]
\]

where:

\[
g(x) = s(x) \otimes h(x)
\]

\[
H_o = \int h(x_o)dx_o
\]

In order to get the iterative form of the parameters, the conjugate gradient method is used to optimize the computation \(^{[10]}\). It can get:

\[
\Theta^{k+1} = \Theta^k + \alpha \tau^k
\]

Where \( \alpha \) is the optimal value of the step width, \( \tau^k \) is the direction of the kth iteration.

\[
\tau^k = \gamma^k + \beta^{k-1} \tau^{k-1}
\]

where \( \gamma^k \) is the gradient of the likelihood function

\[
\gamma^k = \nabla_{\Theta^k} L^k
\]

When \( k = 0 \), \( \beta^{k-1} \) is not existed, so

\[
\tau^k = \gamma^k
\]

When \( k \neq 0 \)

\[
\beta^{k-1} = \frac{\langle \gamma^k - \gamma^{k-1}, \gamma^k \rangle}{\langle \gamma^{k-1}, \gamma^{k-1} \rangle}
\]

the value of the step width \( \alpha \) is the one that maximize the \( L(\Theta^k + \alpha \tau^k) - L(\Theta^k) \). These steps are keeping on, until
\[ \left\| (L^k - L^{k-1}) / L^k \right\| < \varepsilon \] (16)

\( \varepsilon \) is an arbitrarily small positive tolerance.

From equation (4), the derivation of the parameter is:

\[ \frac{\partial L}{\partial \theta_i} = -\int [h_{\theta_i} \otimes s - g * \hat{h}_{\theta_i} \otimes \hat{s}]dx_i \] (17)

where

\[ \hat{h}_{\theta_i} = \frac{\partial \hat{h}}{\partial \theta_i} \]

(18)

from Equation (5), it can get:

\[ \hat{h}_\sigma = -\frac{1}{\pi \sigma^3} \exp[-\frac{X^2}{2\sigma^2}] + \frac{X^2}{2\pi \sigma^5} \exp[-\frac{X^2}{2\sigma^2}] \] (19)

In the three-dimensional coordinate system, the form of the PSF is:

\[ h_{\sigma_1,\sigma_2} = \frac{1}{2\pi \sigma_1^2} \cdot \frac{1}{\sqrt{2\pi \sigma_2}} \exp[-\frac{x^2 + y^2}{2\sigma_1^2} - \frac{z^2}{2\sigma_2^2}] \] (20)

Where \( \sigma_1 \) is the variance of the x and y direction, and \( \sigma_2 \) is the variance of the z direction. So from the Equation (18), the Equation (19) should be replaced by:

\[ \hat{h}_\sigma = -\frac{1}{\pi \sigma_1^3} \cdot \frac{1}{\sqrt{2\pi \sigma_2}} \exp[-\frac{x^2 + y^2}{2\sigma_1^2} - \frac{z^2}{2\sigma_2^2}] \]

\[ \begin{aligned}
&+ \frac{x^2 + y^2}{2\pi \sigma_1^5} \cdot \frac{1}{\sqrt{2\pi \sigma_2}} \exp[-\frac{x^2 + y^2}{2\sigma_1^2} - \frac{z^2}{2\sigma_2^2}] \\
&\hat{h}_\sigma = \frac{1}{2\pi \sigma_1^2} (-\frac{1}{\sqrt{2\pi \sigma_2^2}}) \exp[-\frac{x^2 + y^2}{2\sigma_1^2} - \frac{z^2}{2\sigma_2^2}] \\
&+ \frac{1}{2\pi \sigma_1^2} \cdot \frac{z^2}{\sqrt{2\pi \sigma_2^4}} \exp[-\frac{x^2 + y^2}{2\sigma_1^2} - \frac{z^2}{2\sigma_2^2}] 
\end{aligned} \] (21)

3. EXPERIMENTAL RESULT

3.1 Two-dimensional image

At the beginning, we use the two-dimensional images to observe the effect of this algorithm.

Firstly, we use the variance equals to 3 and 7*7 Gauss model to convolve with the image (figure 1) that will get the blurred image (figure 2). Secondly, we use this PBD algorithm to recover it. In the experiment, we estimate the
original variance of the Gauss model equals to 5 at first, and then we follow the specimen’s iterative formula to update
the specimen function. After finishing updating the specimen function, we set the step width $\alpha = 0.1$, set the initial
variance $\sigma = 0$. During the iteration, we should find a variance that will maximize the likelihood function. Let’s
update the variance of the Gauss function to be the new one. We repeated this process for about the 100 times and the
recovery image will be fine.

Figure 1: original image  Figure 2: blurred image with $\sigma = 3.0$  Figure 3: recovered image $\sigma = 4.0$

Figure 4: recovered image $\sigma = 5.0$  Figure 5: recovered image $\sigma = 6.0$

Figure 3 is the image after using this PBD algorithm to Figure 2 whose initial estimated variance $\sigma = 4.0$. Figure
4 is the image after using PBD algorithm to figure 2 whose initial estimated variance.

Figure 6: $\sigma = 4.0$. (a) Every variance of Gauss function after updating. (b) Every log-likelihood function value after updating.
Figure 7: $\sigma = 5.0$. (a) Every variance of Gauss function after updating. (b) Every log-likelihood function value after updating.

Figure 8: $\sigma = 6.0$. (a) Every variance of Gauss function after updating. (b) Every log-likelihood function value after updating.

$\sigma = 5.0$. Figure 5 is the image after using PBD algorithm to figure 2 whose initial estimated variance $\sigma = 6.0$. Each of these three estimations is able to finely recover the blurred image.

Figures 6-8 show the correspondent variance and log-likelihood function values of Figures 3-5. The results show that after 10 times iterations, these values will be approximate to a constant and they are also very close to the real one. So we can get the satisfactory results.

Figures 9-11 are another group of example. Figure 9 is the original image. Figure 10 is the blurred image using $\sigma = 3.0$ and 7*7 Gauss model. After the PBD, the edges of the objects are cleared on the large part of the image. But it still has some problems. There are some ripples happened around the high frequency parts. This phenomenon is the result of the expectation maximization algorithm (EM). So we can’t avoid it until we use another algorithm.
From these two two-dimensional examples, we can deduce that for some images, the PBD algorithm can do very well. But for some images it can’t work well.

### 3.2 Three-dimensional image

In the biological field, most images recorded by the microscope are three-dimensional images. Figure 12 is a serial of optical slices that are randomly chose from 50 images. Figure 13 is the recovery effect of the Figure 12’s sequence. After the PBD, the high frequency parts of the object are sharpened, so there is much more useful information can be detected. While the low frequency parts of the object are still unseen, so can still hide some useless information. As above-mentioned, this algorithm can help us to detect the important information of the biological specimen without clearing the non-important information. Figure 14 shows the variance and log-likelihood function value after each updating. Roughly after 10 times, the result will access to a robust value.
Figure 13: the recovery sequence of the optical slices. (a) is the 15th image, (b) is the 17th image, (c) is the 18th image, (d) is the 20th image, (e) the 22nd image, (f) is the 24th image, (g) is the 27th image, (h) is the 30th image.

Figure 14: (a) Every variance of Gauss function after updating. (b) Every log-likelihood function value after updating.

4. CONCLUSION

In this algorithm we have to know the rough range of the initial variance of the Gauss function that was used to blur the image firstly. If we don’t know it, we won’t get the accurately estimation of the variance for the first iteration, and as a matter of fact we won’t get the ideal result of it. If our first estimation of the variance is less or much larger than the actual one, it will violate the condition of the unique, which means we can’t accurately estimate the variance, some times the result is not convergent. So it won’t give us the ideal result. Sometimes, if the image has already been blurred by an unknown variance, we actually couldn’t recover the image rightly.

With our future work, we can test more parameters for this algorithm. And we can also test the DV (depth variance) algorithm combined with PBD algorithm to recover the sequence images of the optical slices.
ACKNOWLEDGEMENTS
This paper is supported by Natural Science Foundation of China (60372079).

REFERENCE