The Study of the Proportion Image of Hyperspectral image

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ABSTRACT

Hyperspectral image can be analyzed by Convex Geometry Analysis (CGA) method. CGA method can unmix endmembers from hyperspectral image. The endmember proportions of mixed pixels can be calculated in inherent dimensional space, and a proportion image, which is called inherent proportion image, is obtained. The endmember proportions of mixed pixels can be calculated in n-space by the Constrained Least Squares, and a proportion image, which is called CLS proportion image, is obtained. In this paper, the inherent proportion image and CLS proportion image of a 30-band remote sensing image are obtained. The two proportion images are similar. The targets that are smaller than earth surface pixel can be identified by the inherent proportion image.

Keywords: Hyperspectral Image, Remote Sensing, Mixed Pixel, Proportion Image, Convex Geometry, Inherent Proportion Image, Endmember Proportions

1. INTRODUCTION

The advent of hyperspectral sensors improves the capability for collection of ground targets in the civil and military fields\textsuperscript{1-3}. Hyperspectral imaging spectrometers collect image cubes containing spectral data reflected from surface substances. Each pixel contains the resultant mixed spectrum from the substances present in the respective pixel spatial coverage\textsuperscript{3}. Past approaches such as mixture modeling unmixing different materials at a sub-pixel level through assuming endmembers and that the spectrum is mainly composed of the endmembers\textsuperscript{4}, and choosing special pixels from the scene as endmembers\textsuperscript{5}. One of the most challenging tasks underlying many hyperspectral imagery applications is the spectral unmixing\textsuperscript{4}.

Hyperspectral unmixing is the procedure by which the endmembers are looked for and chosen from hyperspectral data, and their corresponding area proportion in each mixed pixel is calculated. The corresponding area proportion is called endmember proportion. The answer depends mainly on the substance distribution at each pixel. Linear mixing model holds when substances are surface distributed in the scene and the incident solar radiation is scattered by the surface through a single bounce\textsuperscript{6,7}.

The concept of convexity geometry can be used to great advantage in the analysis of hyperspectral data. Convex and affine geometry in n-dimensions provide powerful tools for the analysis, understanding and visualization of hyperspectral data\textsuperscript{6}. Independent Component Analysis (ICA) has recently been proposed as a tool to unmix hyperspectral data. Hyperspectral data source dependence and its impact on ICA performance\textsuperscript{9,10}. We have had some approaches about spectral unmixing method- Convex Geometry Analysis method\textsuperscript{11}.

In this paper, we introduce the study of the proportion image of hyperspectral image. A 30-channel remote sensing image is analyzed. The endmember proportions of mixed pixels in the image are calculated in...

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*n*-dimensional space by the Constrained Least Squares, and a proportion image, which is called CLS proportion image, is obtained. The endmember proportions of mixed pixels in the remote sensing image are calculated in inherent dimensional space, and a proportion image, which is called inherent proportion image, is obtained. Its inherent proportion image is compared with the CLS proportion image. The two proportion images are similar. The targets that are smaller than earth surface pixel can be identified by the inherent proportion image.

2. LINEAR MIXING MODEL

The linear mixed-pixel problem in remote sensing is the simplest model for analyzing hyperspectral data. The model is to assume that the response vector in *n* spectral channels on any pixel. We testify linear mixing model for spectral reflectance under uniformly illumination. Linear mixing model for spectral reflectance is described \(^4,8,11-14\): mixed pixel spectral reflectance is endmembers spectrum linear addition. Mixed pixel spectral reflectance is called mixed pixel complex spectra. We express linear mixing model by using matrix. Here, we discuss theory value, linear mixing model for spectral reflectance:

\[
\rho = Pf
\]

Where \(m\) is the number of endmembers, \(\rho\) is vector in *n* spectral bands on a mixed pixel, \(P\) is \(n\) by \(m\) matrix whose columns are the endmember spectra, \(f\) is an \(m\) by 1 vector of proportions, \(1_m\) is \(m\) by 1 vector whose all elements are 1. Further more, the proportions is greater than or equal to 0. The linear mixing model is tenable for spectral data too. Spectral reflectance is substituted for corresponding spectral data.

Our purpose is how to unmix the endmembers in hyperspectral image basis linear mixing model (1). After the endmembers are unmixed, we can calculate the endmember proportions of each mixed pixel. The proportion image of hyperspectral image can be obtained.

3. CONVEX GEOMETRY ANALYSIS METHOD\(^1\)

3.1 Inherent dimension space and simplex

Two equations in (1) can be expressed for one matrix equation by operation principle of matrix. The matrix equation (1) can be written as following:\(^1\):

\[
\tilde{\rho} = \tilde{P}f
\]

Where \(\tilde{\rho}\) is \(\rho\) adding one 1 vector, \(n+1\) by 1 vector. Where, \(\tilde{P}\) is called augmented vector of \(\rho\). \(\tilde{P}\) is the \(P\) adding one rank 1, is called augmented matrix of \(P\), \(n+1\) by \(m\) matrix, whose columns are augmented vector \(\tilde{\rho}_j = [\rho_j 1]^T\) \((j = 1, 2, \ldots, m)\) of endmembers spectral vector \(\rho_j (j = 1, 2, \ldots, m)\). Namely \(\tilde{P} = [\tilde{\rho}_1 \ \tilde{\rho}_2 \ \cdots \ \tilde{\rho}_m]\).

If the number of the endmembers is \(m\), we only select \(n=m-1\) bands to form \((m-1)\)-space, which is called inherent dimensional space. The linear mixed problem can be solved in the inherent dimensional space. The \(m-1\) is inherent dimensionality\(^2,11\). Here, \(\tilde{P}\) is an \(m\) square matrix.
If the solution of matrix equation (2) is only, value of corresponding determinant of $|\tilde{P}|$ is not equal to 0. Namely $\tilde{P}$ is nonsingular. Now the $m$ endmembers isn’t coplanar, form the simplest geometric figure, that is nonsingular, is called $(m-1)$-simplex. Where $n=m-1$. Hence condition under which $m$ endmembers is simplex in $(m-1)$-space is

$$|\tilde{P}| \neq 0$$

Then

$$f = \tilde{P}^{-1} \tilde{\rho}^{-1}$$

If $m=3$, the three spectral vector points of the three endmembers, which satisfy (3), aren’t co-linear, form a triangle. Namely 2-simplex is a triangle. Mixed pixels inside the triangle(Figure 1). Other wise the three endmembers are co-planar. If $m=4$, the four spectral vector points of the four endmembers, which satisfy (3), aren’t co-planer, form a tetrahedron. Namely 3-simplex is a tetrahedron. Mixed pixels are inside the tetrahedron(Figure 2 and Figure 3). Otherwise the four endmembers are co-planer. If $m$ is greater than 4, $(m-1)$-simplex is also same. Namely $(m-1)$-simplex satisfy (3). Otherwise the endmembers lie in hyperplane($m>4$). The projection of the $(m-1)$-simplex in reduced dimensional space form a convex body whose vertices are the enmembers or some of the endmembers(Figure 1, Figure 2, and Figure 3.)

**Figure 1.** 2-d simplex  
(a) 2-d simplex is a triangle whose vertices are the endmembers. Mixed pixels inside the triangle. 
(b), (c) The projection of 2-d simplex in 1-d space is a line section whose endpoints are two about three endmembers. Mixed pixels are on the two line sections.

**Figure 2.** 3-simplex  
(a) 3-simplex is a tetrahedron whose vertices are the endmembers. Mixed pixels inside the tetrahedron.  
(b), (c), (d) The projection of 3-simplex in 2-space may be a quadrilateral whose vertices are the endmembers. Mixed pixels are inside the quadrilaterals.
Complex spectra of mixed pixel and endmembers spectrum are related in matrix equation (2). Inherent dimensional space $\tilde{P}$ is an $m$ square matrix, whose columns are $m$ endmembers augmented vectors. All the mixed pixels will be interior of the $(m-1)$-simplex. This result is audio-visual in 2 or 3 dimension space. If some columns of $\tilde{P}$ is changed augmented vectors of some or all mixed pixels, $\tilde{P}$ forms another $m$ square matrix $\tilde{V}$. We use $\| \tilde{P} \|$ to signify absolute value of corresponding determinant of $\tilde{P}$. For any $m$-1 dimension space, there are conclusions as following:

We set symbol: $r_i (l=1,2,\cdots,m)$ are $m$-1 by 1 vectors of the endmembers or mixed pixels, $\rho_j (j=1,2,\cdots,m)$ $m$-1 by 1 vector of an endmember, $\tilde{r}_i = [{r_1^T \ 1}]^T (l=1,2,\cdots,m)$, $\tilde{\rho}_j = [{\rho_j^T \ 1}]^T (j=1,2,\cdots,m)$, $\tilde{V} = [\tilde{r}_1 \ \tilde{r}_2 \ \cdots \ \tilde{r}_m]$, $\tilde{P} = [\tilde{\rho}_1 \ \tilde{\rho}_2 \ \cdots \ \tilde{\rho}_m]$, Then there is conclusion as following:

$$\| \tilde{V} \| \leq \| \tilde{P} \|$$

Equal sign of (5) is only under as $r_i (i=1,2,\cdots,m)$ are $m$ different endmembers. We will unmix endmembers by (5). In inherent dimensional space, the endmembers are mixed pixels that are characterized by

$$\| \tilde{P} \| \rightarrow \text{max}$$

The mixed pixels that are characterized by (6) are the endmembers. The steps of CGA method see literature 11.

3.2 Inherent proportion image

The endmembers are looked for by (6). The endmember proportions of each mixed pixel in hyperspectral image can be calculated by (4) in inherent dimensional space. Proportion image of hyperspectral image is obtained. The proportion image is called inherent proportion image.
3.3 CLS proportion image

The endmember proportions of mixed pixels are calculated in \( n \)-space by the Constrained Least Squares method\(^{14} \), which is as follow:

\[
    f_{\text{CLS}} = \min_{f^T f = 1} \| \rho - Pf \|^2 = \left( I - \left( P^T P \right)^{-1} l_m l_m^T \right) P^T \rho + \left( P^T P \right)^{-1} l_m l_m^T P^T \rho + \left( P^T P \right)^{-1} l_m l_m^T
\]

In \( n \)-space, the endmember proportions are calculated by (7). A proportion image, which is called CLS proportion image, is obtained.

4. ANALYSIS RESULTS AND DISCUSSION

In this paper, we analyze a 30-band remote sensing image with \( CGA \) method. A band of the remote sensing images is shown in figure 4. We obtain four endmembers. They are vegetation (Endmember A), artificiality (Endmember B), concrete and asphaltum (Endmember C) and structure (Endmember D) respectively. Their spectral sign curves are shown in figure 5. Its size is \( 340 \times 340 \) pixels. Size of earth surface pixels is \( 1.5 \) meter \( \times \) \( 1.5 \) meter. Its inherent proportion image (Figure 6.) is obtained. Its CLS proportion image (Figure 7.) is obtained.

Figure 6 and Figure 7 show that the inherent proportion image and the CLS proportion image of the 30-band remote sensing image are similar. We can calculate proportion image with inherent channels in inherent dimensional space. Hyperspectral data can be analyzed in inherent dimensional space. Analysis in inherent dimensional space gives a more thorough understanding of the hyperspectral data.
5. CONCLUSIONS

Convex geometry provides powerful tools for the analysis, understanding and visualization of hyperspectral data. In inherent dimensional space, the endmembers are found. The inherent image of hyperspectral image can be obtained. Hyperspectral data can be analyzed in its inherent dimensional space. Inherent proportion image of hyperspectral data can identify target that is smaller than earth pixel.

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REFERENCES


