The Selection of Inherent Channels of Hyperspectral Data with Volume Method
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ABSTRACT
We analyze the inherent channels of hyperspectral data with convex geometry analysis method. In this paper, a method Volume Method, which selects the inherent channels of hyperspectral data, is presented. The concept of convexity geometry can be used to great advantage in the analysis of hyperspectral data. Convex simplex and inherent dimensionality concept is discussed on base of convex geometry. A set of 252-band hyperspectral data is applied to testify the Volume Method. The endmember proportions are calculated in the inherent dimensional space whose channels are selected by the Volume Method, compared with Constrained Least Squares Method in 252-space.

Keywords: Hyperspectral Data, Inherent Channels, Endmember, Endmember Proportions

1. INTRODUCTION
Aircraft and satellite photography is well-established technology for remote sensing. Hyperspectral data can be acquired, however, is relatively new. Hyperspectral data are capable of precisely capturing narrow bands of spectra through a wide range of wavelengths.

One of the most challenging task underlying many hyperspectral data applications is the spectral unmixing, which decomposes a mixed pixel into a collection of reflectance spectra, called endmember signatures, and their corresponding fractional abundance\textsuperscript{1}. Convex geometry analysis(CGA)\textsuperscript{2} method and vertex component analysis(VCA)\textsuperscript{1} method can unmix endmembers. N-dimensional spectral analysis seeks to determine the inherent dimensionality of a data set and to analyze it in its native dimensionality\textsuperscript{1}. If the endmembers are known or are looked for by CGA\textsuperscript{2} and VCA\textsuperscript{1} method, we can analyze hyperspectral data in inherent dimensionality space. One of the most challenging tasks underlying many hyperspectral data applications is how to select inherent channels from hundreds of spectral channels. In this paper, we present a method Volume Method, which selects the inherent channels of hyperspectral data. The Volume Method can select the inherent channels from hundreds of spectral channels, which give the remarkable spectral characteristic of the endmembers.

About the selection of inherent channels with Volume Method, our past approach\textsuperscript{2} was introduced simply only. Its proof was not given\textsuperscript{2}. In this paper, its demonstration is given.

2. LINEAR SPECTRAL MIXTURE MODEL
Linear spectral mixture model is testified for spectral reflectance and spectral data under uniformly illumination. Linear mixing model for spectral reflectance and spectral data is described\textsuperscript{2-6}: mixed pixel spectral reflectance is endmembers spectrum linear addition. Mixed pixel spectral reflectance and spectral

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data is called mixed pixel complex spectra. We express the linear mixing model by using matrix. Here, linear mixing model for spectral reflectance and spectral data:

\[
\rho = Pf \\
1_m^T f = 1
\]

(1)

Where \( m \) is the number of endmembers, \( \rho \) is vector in \( n \) spectral bands on a mixed pixel, \( P \) is \( n \) by \( m \) matrix whose columns are the \( m \) endmembers spectra, \( f \) is an \( m \) by 1 vector of proportions, \( 1_m \) is \( m \) by 1 vector whose all elements are 1. Further more, the proportions is greater than or equal to 0.

One of the most challenging tasks underlying many hyperspectral data applications is the spectral unmixing. Our purpose is how to analyze hyperspectral data on basis of the linear mixing model (1).

3. INHERENT DIMENSIONALITY

If the kind of ground objects to identify is \( m \), \( m-1 \) spectral channels are selected from the \( n \) spectral channels of hyperspectral data. The \( m-1 \) spectral channels can analyze hyperspectral data. Hyperspectral data of the \( m-1 \) spectral channels are points of a \((m-1)\)-space, whose any coordinates axis the \( m-1 \) spectral channels. (2) and (3) give linear spectral mixture model in the \((m-1)\)-space. So, \( m-1 \) is called inherent dimensionality. The \( m-1 \) spectral channels are called inherent channels.

\[
\rho(\lambda_i) = F_1 \rho(\lambda_1) + F_2 \rho(\lambda_2) + \cdots + F_m \rho(\lambda_m) \\
\rho(\lambda_{m-1}) = F_1 \rho(\lambda_{m-1}) + F_2 \rho(\lambda_{m-1}) + \cdots + F_m \rho(\lambda_{m-1}) \\
\]

(2) and (3) give:

\[
\begin{bmatrix}
\rho(\lambda_1) \\
\rho(\lambda_2) \\
\vdots \\
\rho(\lambda_{m-1}) \\
1
\end{bmatrix}
= 
\begin{bmatrix}
\rho(\lambda_1) & \rho_2(\lambda_1) & \cdots & \rho_m(\lambda_1) \\
\rho(\lambda_2) & \rho_2(\lambda_2) & \cdots & \rho_m(\lambda_2) \\
\vdots & \vdots & \ddots & \vdots \\
\rho(\lambda_{m-1}) & \rho_2(\lambda_{m-1}) & \cdots & \rho_m(\lambda_{m-1}) \\
1 & 1 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_m
\end{bmatrix}
\]

(4)

set,

\[
\tilde{\rho} = 
\begin{bmatrix}
\rho(\lambda_1) \\
\rho(\lambda_2) \\
\vdots \\
\rho(\lambda_{m-1}) \\
1
\end{bmatrix},
\tilde{P} = 
\begin{bmatrix}
\rho(\lambda_1) & \rho_2(\lambda_1) & \cdots & \rho_m(\lambda_1) \\
\rho(\lambda_2) & \rho_2(\lambda_2) & \cdots & \rho_m(\lambda_2) \\
\vdots & \vdots & \ddots & \vdots \\
\rho(\lambda_{m-1}) & \rho_2(\lambda_{m-1}) & \cdots & \rho_m(\lambda_{m-1}) \\
1 & 1 & \cdots & 1
\end{bmatrix}
\]

(5)

Where \( \tilde{\rho} \) is \( \rho \) adding one 1 vector, \( m \) by 1 vector, \( \tilde{P} \) is called augmented vector of \( \rho \). \( \tilde{P} \) is the \( P \) adding one rank 1 , is called augmented matrix of \( P \). \( \tilde{P} \) is \( m \) by \( m \) matrix, whose columns are augmented.
vector \( \tilde{\rho}_j = [\rho_j^T \ 1]^T (j = 1, 2, \cdots, m-1) \) of endmember spectral vector \( \rho_j (j=1,2,\cdots,m-1) \). Namely, \( \tilde{P} = [\tilde{\rho}_1 \ \tilde{\rho}_2 \ \cdots \ \tilde{\rho}_m] \) \( \tilde{P} \) is said to be a square matrix of order \( m \). Here, (2) and (3) are given as follows: 

\[
\tilde{\rho} = \tilde{P} f
\]  

(6)

If \( \tilde{P} \) is a square matrix, the determinant whose elements are the same, and in the same positions as the elements of \( \tilde{P} \), is said to be the determinant of the matrix \( \tilde{P} \); it is denoted by \( |\tilde{P}| \). If \( \rho \) and \( f \) are one to one correspondence, \( |\tilde{P}| \) isn’t zero. If \( |\tilde{P}| \neq 0 \), \( \tilde{P} \) is full rank. When \( m \) endmembers form simplex in \( (m-1) \)-space, the corresponding \( \tilde{P} \) of \( m \) endmembers is characterized by

\[
|\tilde{P}| \neq 0
\]  

(7)

When the corresponding \( \tilde{P} \) of \( m \) endmembers is characterized by (7), the endmembers form simplex in \( (m-1) \)-space. The simplex in \( (m-1) \)-space is called \( (m-1) \)-simplex. Here, \( \tilde{P} \) is invertible matrix. So that

\[
f_{\text{val}} = \tilde{P}^{-1} \tilde{\rho}
\]  

(8)

Where the subscript \( \text{val} \) on \( f \) represents the endmember proportions that are calculated in inherent dimensional space. When \( m=3 \), the corresponding \( \tilde{P} \) of the three endmembers satisfy (7). The corresponding spectral vector points of the endmembers aren’t co-linear, form a triangle. Namely 2-simplex is a triangle. Mixed pixels inside the triangle(Figure 1.). Other wise the three endmembers are co-linear. When \( m=4 \), the corresponding \( \tilde{P} \) of of the four endmembers which satisfy (7). The corresponding spectral vector points of the endmembers aren’t co-planers, form a tetrahedron. Namely simplex in 3-space is a tetrahedron. Mixed pixels inside the tetrahedron(Figure 2.). Other wise the four endmembers are co-planer. If \( m \) is greater than 4, simplex in \( (m-1) \)-space is also same. Namely \( (m-1) \)-simplex satisfy (7). Other wise the endmembers lie in a hyperplane(\( m>4 \)).

![Figure 1. 2-simplex is a triangle. Mixed pixels inside the triangle](image)

### 4. Selecting Inherent Channels by Volume Method

When the endmembers are given, how to select \( m-1 \) channels from all channels of hyperspectral data is a committed step. If the \( m-1 \) channels form inherent dimensional space, (7) must be satisfied. Otherwise, the selected \( m-1 \) channels are coherent for the endmembers. In other word, the \( m-1 \)
channels aren’t inherent. $|P|$ expresses absolute value of corresponding determinant of $\tilde{P}$.

According to knowledge of geometry, $|P|$ is direct ratio to area or volume of corresponding simplex whose vertices is the $m$ endmember vector points in 2-space or 3-space.

Figure 2. 3-simplex is a tetrahedron. Mixed pixels are inside the tetrahedron

For three endmembers, inherent dimensional space is 2-space. The greater $|\tilde{P}|$, the area of simplex (triangle, Figure 1.), whose vertex are the three endmembers, will possess greater value. In other word, the larger $|\tilde{P}|$, the longer distance of separation of the three endmembers will possess. For four endmembers, inherent dimensional space is 3-space. If $|\tilde{P}|$ is greater, the volume of simplex (tetrahedron, Figure 2.), whose vertex are the four endmembers, will possess greater value. In other word, the greater $|\tilde{P}|$, the longer distance of separation of the four endmembers will possess.

Higher dimensional space is the same as 2-space or 3-space. In higher dimensional space, $|\tilde{P}|$ can represent level of separation of the $m$-1 channels for the endmembers. $|\tilde{P}|$ is direct ratio to the volume of corresponding simplex whose vertices is the $m$ endmember vector points in higher dimension($m$>4) space. The greater $|\tilde{P}|$, the longer distance of separation of the $m$-1 endmembers will possess. Namely, dispersion of the endmembers in the inherent dimensional space, which is formed by the $m$-1 channels, is greater. The best inherent channels are selected according to the value of $|\tilde{P}|$. The best inherent channels are selected by

$$|\tilde{P}| \rightarrow \text{max} \quad (9)$$
The method that selects inherent channels with (9) is called Volume Method.

5. EXPERIMENT RESULTS AND DISCUSSION

We did experiment to testify the Volume Method. The number of endmember is four. The spectral curves of the four endmembers and a mixed pixel are shown in Figure 3. The numbers of spectral channel is 252. Inherent channels are selected with Volume Method (Figure 3.) Data of the inherent channels are listed in table 1. Simplex in 4-d inherent dimensional space is shown in Figure 4.

![Figure 3. Four endmembers and mixed pixel spectral curve and inherent channels are selected with volume method.](image)

The endmember proportions of the mixed pixel is calculated in inherent dimensional space by (8) as follow:

$$f_{\text{val}} = \begin{pmatrix} 0.219 \\ 0.154 \\ 0.314 \\ 0.313 \end{pmatrix}$$

The endmember proportions of each mixed pixel can be calculated with Constrained Least Squares that is given as follow:

$$f_{\text{CLS}} = \min_{f_{\lambda} \in \mathbb{R}^m} \left\| \rho - Pf \right\|^2 = \left( I - \left( \frac{(P^TP)}{1_m} \right)^{-1} I_m \right) \left( \frac{(P^TP)}{1_m} \right)^{-1} P^T \rho + \frac{\left( \frac{(P^TP)}{1_m} \right)^{-1} I_m}{\left( \frac{(P^TP)}{1_m} \right)^{-1} I_m}$$

Where $P$ and $\rho$ is vector in 252-space.

The endmember proportions of the mixed pixel is calculated in 252 dimensional space by (11) as follow:
\[ f_{\text{CLS}} = (0.2200, 0.1510, 0.3050, 0.324) \]  
(12)

(11) and (12) give:
\[ \left\| f_{\text{val}} - f_{\text{CLS}} \right\| \approx \frac{0.01456}{0.5189} \approx 0.0280 \approx 3\% . \]  
(13)

Figure 3 shows spectral characteristics of the endmembers are notable in inherent channels that are selected with the volume method. Data in table 1 shows that the deference among spectral data of the endmembers in the inherent channels is greater. Figure 3 shows that spectral characteristic peaks are given in the inherent channels that are selected with the volume method, and the characteristic peaks mixed pixel are given in the inherent channels too. The distance among the inherent channels is further. From (13), the spectral characteristics of mixed pixel are given in inherent dimensional space that is selected with the volume method. We can analyze mixed pixel problem in inherent dimensional space. It is needn’t to analyze mixed pixel problem in \( n \)-space. Where \( n \) is the number of hyperspectral data channels.

Table 1: Inherent channels that are selected with the volume method and spectral data of the endmembers in the inherent channels

<table>
<thead>
<tr>
<th>波长(nm)</th>
<th>反射率(nm)</th>
<th>Endmember A</th>
<th>Endmember B</th>
<th>Endmember C</th>
<th>Endmember D</th>
<th>mixed pixel</th>
</tr>
</thead>
<tbody>
<tr>
<td>536.1</td>
<td>1</td>
<td>0.0471</td>
<td>0.1373</td>
<td>0.0699</td>
<td>0.2909</td>
<td></td>
</tr>
<tr>
<td>618.1</td>
<td>0.8143</td>
<td>0.9332</td>
<td>0.046</td>
<td>0.0467</td>
<td>0.3511</td>
<td></td>
</tr>
<tr>
<td>951.9</td>
<td>0.8069</td>
<td>0.7989</td>
<td>0.2011</td>
<td>0.6667</td>
<td>0.5714</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Simplex of the four endmembers in the inherent dimensional space, whose inherent channels are selected with the volume method, is a tetrahedron. Each mixed pixel is inside the tetrahedron. The two tetrahedrons in (a) and (b) are the simplexes that are looked from two different viewpoints.

6. CONCLUSIONS

Spectral characteristics of the endmembers are notable on inherent channels, which are selected with the volume method. The deference among spectral data of the endmembers in the inherent channels is greater.

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Spectral characteristic peaks can be given in the inherent channels that are selected with the *volume method*. Mixed pixel problem can be solved in inherent dimensional space. It is neend’t to analyze mixed pixel problem in *n*-d space. Where *n* is the number of hyperspectral data channels. Inherent channels that are selected by *volume method* can give spectral characteristics of the endmembers and mixed pixels.

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