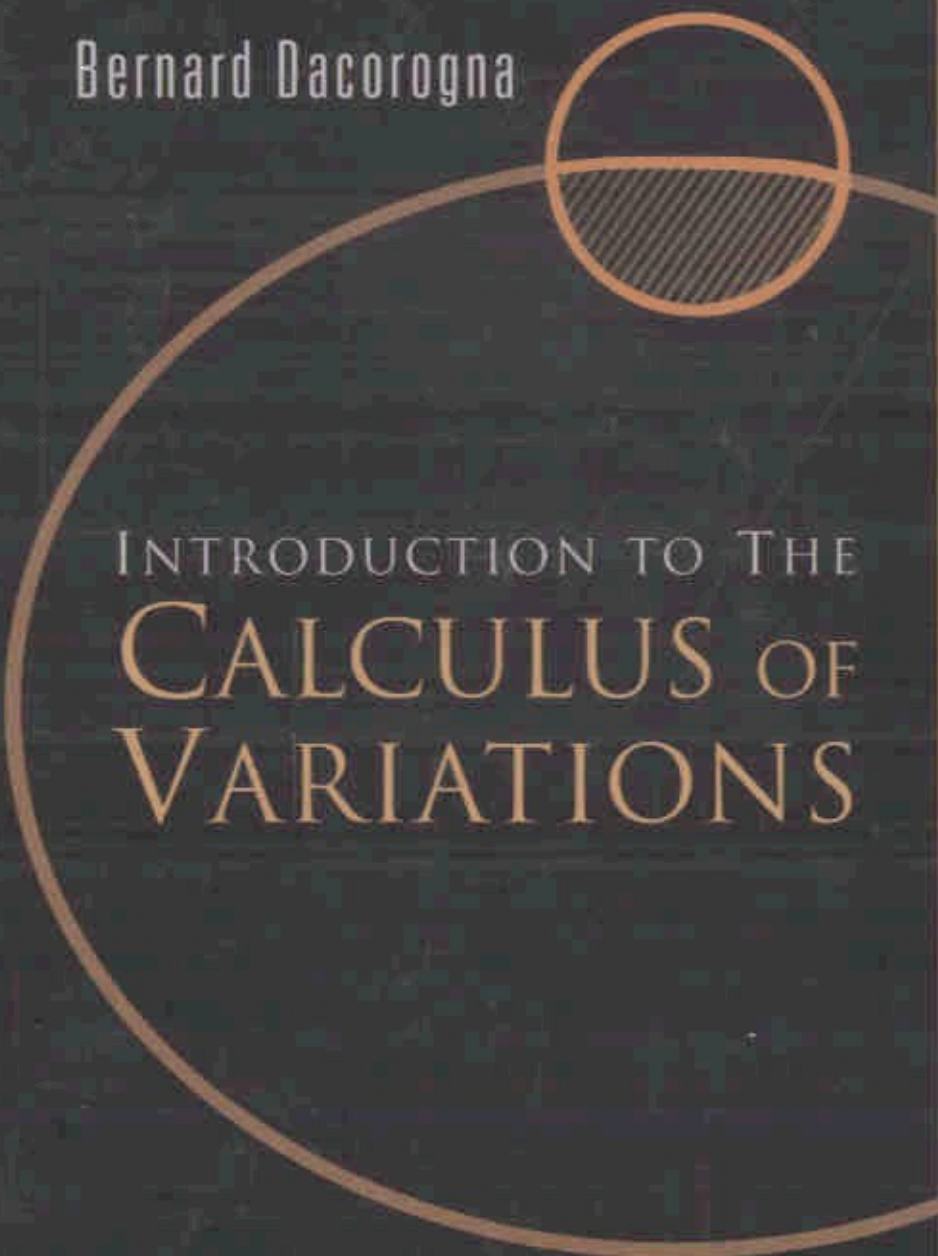


Bernard Dacorogna



INTRODUCTION TO THE
CALCULUS OF
VARIATIONS

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