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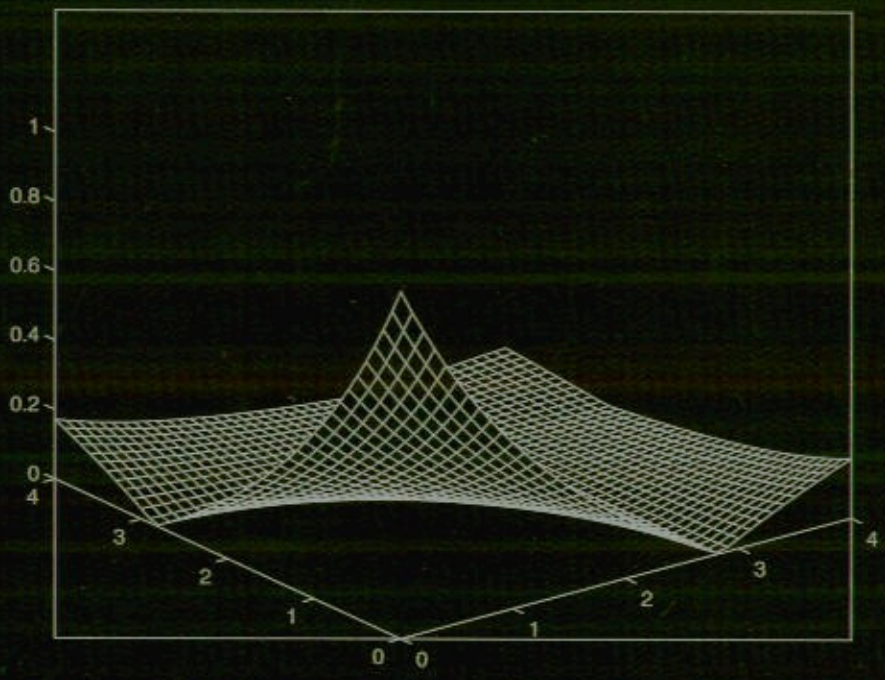
A Tutorial on Elliptic PDE Solvers and Their Parallelization



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