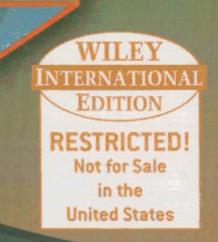
6th Edition

## Design and Analysis of Experiments



Douglas C. Montgomery

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