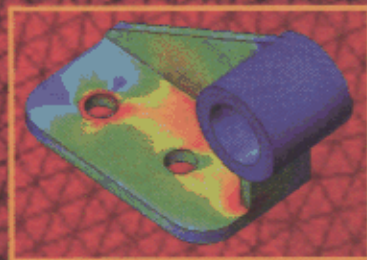
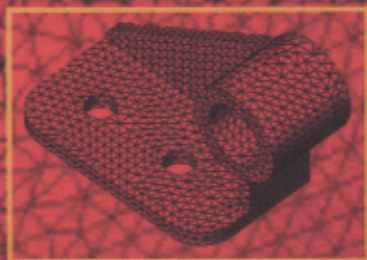
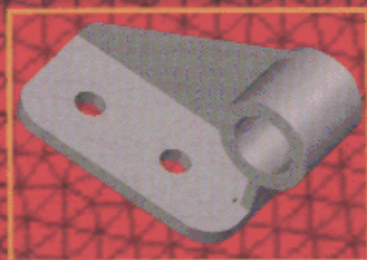


J.E. AKIN

FINITE ELEMENT ANALYSIS WITH ERROR ESTIMATORS



**AN INTRODUCTION TO THE FEM AND
ADAPTIVE ERROR ANALYSIS FOR ENGINEERING STUDENTS**

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