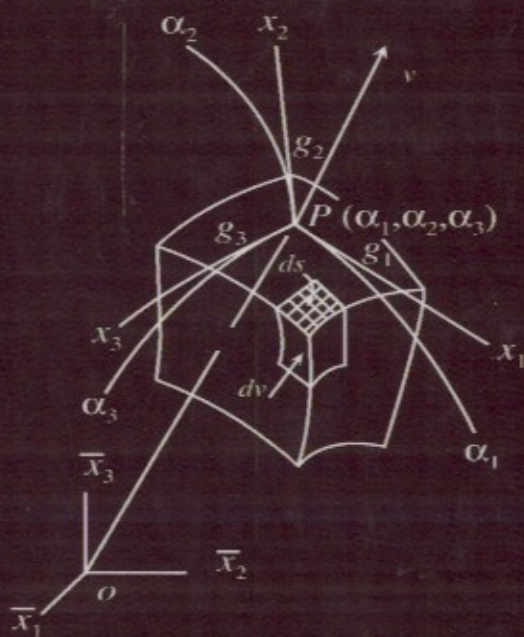


CRC SERIES: MODERN MECHANICS AND MATHEMATICS

Hybrid and Incompatible Finite Element Methods



Theodore H.H. Pian
Chang-Chun Wu



Chapman & Hall/CRC
Taylor & Francis Group

Contents

Chapter 1 Variational Formulation of Finite Element Methods in Solid Mechanics

1.1	Introduction	1
1.2	Equations for 3-D Elasticity	2
1.3	Conventional Variational Principles in Solid Mechanics	4
1.4	Modified Variational Principles for Relaxed Continuity or Equilibrium Conditions along Interelement Boundaries.....	5
1.5	Assumed Displacement Finite Elements	6
1.6	Assumed Stress Hybrid Finite Elements.....	10
1.6.1	Formulation Based on the Principle of Minimum Complementary Energy	10
1.6.2	Elements with <i>a priori</i> Satisfaction of Equilibrium and Compatibility Conditions	11
1.6.2.1	Hybrid Trefftz Elements	11
1.6.3	Formulation Based on the Hellinger–Reissner Variational Principle.....	13
1.7	Hybrid Strain Finite Elements.....	15
1.8	Hybrid Finite Elements by the Hu–Washizu Principle	15
1.9	Hybrid Displacement Finite Elements.....	17
	References	17

Chapter 2 Foundation of Incompatible Analysis

2.1	Introduction	21
2.2	Energy Inequality and Elliptic Conditions.....	21
2.3	Weak Connection Condition of Incompatible Elements.....	25
2.4	Numerical Stability of Incompatible Elements.....	29
2.5	Consistency and Patch Test Condition (PTC)	33
2.6	Generation of Incompatible Functions: General Formulation	35
2.7	Relaxation of the PTC: The Revised-Stiffness Approach.....	38
2.8	The PTC in Curvilinear Coordinates	40
2.9	Equivalent Nodal Load and Recovery of Stress	45
	References	48

Chapter 3 Incompatible Elements for the Theory of Elasticity

3.1	Introduction.....	51
3.2	Four-Node Plane Incompatible Elements: NQ6.....	51
3.3	P_2 -Linked Incompatible Methods with the Fewest Degrees of Freedom (DOF).....	57
3.4	Eight-Node 3-D Solid Incompatible Elements.....	63
3.5	Axisymmetric Incompatible Elements.....	67
3.5.1	Patch Test Condition.....	67
3.5.2	Revised-Stiffness Approach.....	70
3.5.3	Numerical Test: Axisymmetric Stress Analysis and Incompressible Calculation.....	73
3.6	Hermite Type Incompatible Plate Elements.....	74
3.6.1	Hermite Type Lateral Displacement Field.....	75
3.6.2	Incompatible Rotation Field.....	77
3.6.3	Element Construction.....	78
3.6.4	Numerical Examples.....	79
3.6.4.1	Patch Test.....	79
3.6.4.2	Bending Circular Plates.....	80
3.7	Bending Model under Reasonable w - θ Constraint.....	81
3.7.1	General Formulation of R-M Plate Elements.....	82
	References.....	88

Chapter 4 Foundation in Mechanics of Hybrid Stress Elements

4.1	Introduction.....	89
4.2	Energy Consistency Analysis for Incompatible Hybrid Elements.....	89
4.3	Patch Test and Element Optimization Condition (OPC).....	91
4.4	Optimization Method for Hybrid Stress Finite Elements.....	93
4.5	Matching Multivariable Parameters.....	96
4.5.1	Situation I.....	97
4.5.2	Situation II.....	98
4.5.3	Situation III (Optimal Parameter Matching).....	98
	References.....	98

Chapter 5 Optimization of Hybrid Stress Finite Elements

5.1	Four-Node Plane Hybrid Element.....	101
5.2	Penalty Equilibrium Hybrid Element P-S(α).....	106
5.3	Three-Dimensional Body 18β -Optimization Hybrid Element.....	112
5.4	Axisymmetric 8β -Optimization Hybrid Element.....	116
5.5	Model Optimization of Hybrid Stress General Shell Element.....	122
5.5.1	Introduction.....	122
5.5.2	The Optimizing Condition and the Variational Principle.....	122
5.5.3	The Discrete Formulation of the Energy Functional.....	125
5.5.4	The Optimizing Formulation of Element Stresses.....	127
5.5.5	Numerical Examples and Results.....	133

5.5.5.1	Thick and Thin Plates	133
5.5.5.2	Locking Test.....	133
5.5.5.3	Thin Rhombic Plates.....	133
5.5.5.4	Pinched Cylinder Problem	135
5.5.5.5	Scordelis Cylindrical Shell.....	136
5.6	Appendix.....	139
5.6.1.	$\overline{\Phi}_I$ and $\overline{\Phi}_{II}$	139
5.6.2.	The Geometry Matrices \overline{B}_q and \overline{B}_λ	140
References	141

Chapter 6 Numerical Stability: Zero Energy Mode Analysis

6.1	Introduction.....	143
6.2	Definition of ZEM	143
6.3	Rank Conditions for Two-Field Mixed-Hybrid Elements.....	144
6.4	Determination of the Zero Energy Modes	147
6.5	Control of the Zero Energy Displacement Modes.....	149
6.6	Control of the Zero Energy Stress Modes.....	150
6.7	Patch Stability Test.....	152
6.8	Examples	154
References	157

Chapter 7 Plastic Analysis of Structures

7.1	Introduction.....	159
7.2	Form of Incompressible Elements and Analysis of Plane Stress Plastic Analysis.....	159
7.3	Incompatible Elements in Plasticity Analysis.....	162
7.3.1	Introduction.....	162
7.3.2	Incompressibility of the Isoparametric Element (Elastic Case).....	163
7.3.3	Incompatible Discrete Model	166
7.3.4	Application to the Limit Analysis.....	169
7.3.4.1	Example I: Thick Elastic-Plastic Cylinder under Internal Pressure	170
7.3.4.2	Example II: Elastic-Plastic Square Tube with a Circular Hole under Internal Pressure.....	171
7.3.4.3	Example III: Elastic-Plastic Tension Specimen with Semicircular Edge Notches.....	172
7.4	Deviatoric Hybrid Model for the Incompressible Medium	174
7.4.1	Introduction.....	174
7.4.2	Deviatoric Variational Principle	175
7.4.3	Three-Field Hybrid Element Formulation.....	177
7.4.4	Incompressible Elimination at the Element Level.....	180
7.4.5	Hybrid Model.....	182
7.4.6	Suppression of the ZEM	183
7.4.7	Numerical Examples.....	185

7.4.7.1	Constant Stress Patch Test	186
7.4.7.2	Numerical Stability Tests	187
7.4.7.3	Plastic Analysis	189
7.4.7.4	Stokes Flow Analysis	189
7.4.8	Conclusion	191
References	193

Chapter 8 Computational Fracture

8.1	Introduction	197
8.2	Dual Path-Independent Integral and Bound Theorem	197
8.2.1	Dual Path-Independent Integral	197
8.2.2	Bound Theorem	198
8.3	Numerical Strategy and Error Measure	200
8.3.1	Numerical Strategy	200
8.3.2	Error Measure	202
8.4	Numerical Tests of Crack Estimation	203
8.5	Incompatible Numerical Simulation of an Axisymmetric Cracked Body	204
8.5.1	Introduction	204
8.5.2	Incompatible Axisymmetric Element AQ6	208
8.5.3	Elastic Solution	211
8.5.4	Elastic-Plastic Solution	213
8.5.5	Plastic Limit Analysis	215
8.5.6	Conclusion	217
8.6	Extension of J to Dynamic Fracture of a Functional Graded Material	217
8.6.1	Introduction	217
8.6.2	The Extended Dynamic J Integral for FGMs	218
8.6.3	The Element-Free Galerkin Method	221
8.6.4	Collocation Method for Coupling the Finite Element and the EFG Methods	223
8.6.5	Numerical Implementation of the Boundary Value Problem	224
8.6.6	Numerical Examples	226
8.6.7	Conclusion	228
8.7	Evaluation of Electromechanical Crack Systems	229
8.7.1	Introduction	229
8.7.2	Dual Integrals for Electromechanical Systems	230
8.7.3	Bound Theorem	231
8.7.3.1	Lower Bound Theorem for $J(u_i, \phi)$	231
8.7.3.2	Upper Bound Theorem for $I(\sigma_{ij}, D_i)$	232
8.7.4	A Limitation on the Bound Analysis	233
8.7.5	Piezoelectric Finite Elements	234
8.7.6	Dual Error Measure	237
8.7.7	Numerical Example	246

8.7.8 Conclusion.....	248
References	249

Chapter 9 Computational Materials

9.1 Hybrid Element Analysis of Composite Laminated Plates.....	253
9.1.1 Introduction.....	253
9.1.2 The State Space and Energy Formulations of Laminated Plates	254
9.1.3 The Laminate Hybrid Element Based on the State Space	257
9.1.4 Implementation of the Interface Stress Continuity	260
9.1.5 Numerical Examples.....	261
9.1.6 Conclusion.....	266
9.2 Bimaterial Interface Hybrid Element for Piezoelectric Laminated Analysis.....	266
9.2.1 Introduction.....	266
9.2.2 Electro-Elastic Variational Formulation and the Governing Equations.....	267
9.2.3 Element Formulations for the 3-D Hybrid Model	272
9.2.4 Interface and Surface Elements	275
9.2.5 Numerical Example	276
9.2.6 Conclusion.....	281
9.3 Numerical Solutions on Fractures of Piezoelectric Materials	283
9.3.1 Introduction.....	283
9.3.2 Constitutive Relations and Variational Functional	285
9.3.3 Piezoelectric Hybrid Element: PZT-Q4	287
9.3.4 Numerical Analyses: Crack-Tip Field Simulation	291
9.3.4.1 Pure Mechanical Loading	291
9.3.4.2 Pure Electric Loading	296
9.3.4.3 Mixed Loading	298
9.3.5 Conclusion.....	301
9.4 Homogenization-Based Hybrid Element for Torsion of Composite Shafts	302
9.4.1 Introduction.....	302
9.4.2 Mathematical Homogenization.....	303
9.4.3 Variational Principles and Finite Elements	308
9.4.4 Displacement-Incompatible Elements	310
9.4.5 Enhanced-Strain Element Based on the Hu–Washizu Principle	313
9.4.6 Enforcing the Periodicity Boundary Condition in the Analysis of the RUC	315
9.4.7 Numerical Examples.....	316
9.4.8 Conclusion and Discussion	323
9.5 A Study of 3-D Braided Piezoceramic Composites	327
9.5.1 Introduction.....	327

9.5.2	Basic Equations.....	328
9.5.3	Asymptotic Expansion	330
9.5.4	Microscale Independent Terms	331
9.5.5	Homogenized Problem	333
9.5.6	Finite Element Variational Statements	336
9.5.7	Finite Element Implementation	337
9.5.8	Numerical Results.....	341
9.5.9	Conclusion.....	342
	References	345
 Chapter 10 Finite Element Implementation		
10.1	Overview.....	351
10.2	Description of Variables and Subroutines.....	351
10.2.1	Definitions of Main Variables and Arrays.....	351
10.2.2	Functions of Subroutines Used.....	353
10.3	Instructions for Input Data	356
10.4	Examples	360
	 Index	 371