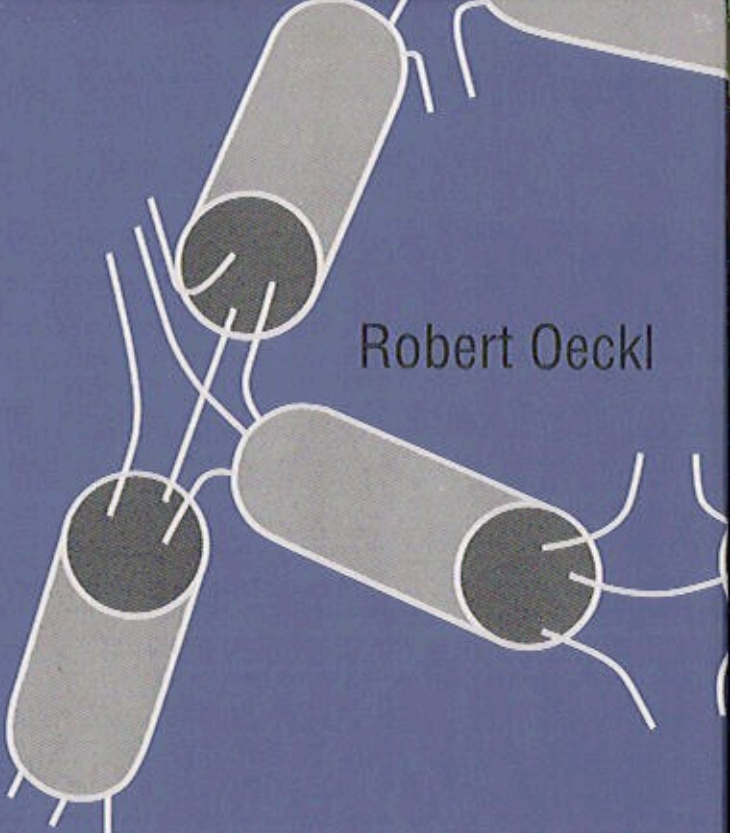




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DISCRETE GAUGE THEORY

FROM LATTICES TO TQFT

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Contents

<i>Preface</i>	v
1. Quantization of Discretized Gauge Theories	1
1.1 Continuum gauge theory	1
1.1.1 Configurations and observables	1
1.1.2 Yang-Mills theory and BF theory	3
1.2 Discretized gauge theory	5
1.2.1 Discretized manifold and connection	5
1.2.2 Yang-Mills theory	7
1.2.3 BF theory	9
1.2.4 BF-Yang-Mills	11
1.3 Path integral quantization	12
1.4 Circuit diagrams – baby version	17
1.4.1 Basics	17
1.4.2 Key identities	18
1.5 Diagrammatic representation	20
1.5.1 Partition function	20
1.5.2 Wilson loop observables	22
1.5.3 Gauge fixing	23
1.6 An exact solution: 2-dimensional lattice gauge theory	25
1.7 Strong-weak duality	27
1.8 The dual of abelian lattice gauge theory	28
2. Topology: Decomposition of Manifolds	33
2.1 Simplicial decompositions	34
2.1.1 Simplices	34

2.1.2	Euclidean complexes	35
2.1.3	Simplicial complexes	35
2.1.4	Triangulations	36
2.1.5	Pachner moves	37
2.2	Cellular decompositions	40
2.2.1	Cells and decompositions	40
2.2.2	Cellular moves	43
3.	Categories and Diagrams	49
3.1	Monoidal categories with structure	49
3.2	Diagrammatic isotopy invariance	57
3.3	Semisimplicity and circuit diagrams	60
3.4	Key identities	66
3.5	Purification	69
4.	Representation Theory: Groups & Hopf Algebras	73
4.1	Groups	73
4.1.1	Representation categories	73
4.1.2	Representative functions	75
4.1.3	Diagrams for representative functions	77
4.1.4	Integration and semisimplicity	77
4.1.5	Recovering the baby diagrams	80
4.2	Hopf algebras and quantum groups	81
4.2.1	Hopf algebras with extra structure	83
4.2.2	Representation categories	86
4.2.3	“Representative functions”	90
4.2.4	Integration and semisimplicity	91
4.2.5	Supergroups	93
4.3	Examples	95
4.3.1	Duality	95
4.3.2	Abelian groups	96
4.3.3	$SU(2)$	97
4.3.4	Supergroups	98
4.3.5	q -deformations	100
5.	Cellular Gauge Theory	105
5.1	Symmetric cellular gauge theory	105
5.2	Non-symmetric cellular gauge theory	109

5.2.1	Dimension 2 – pivotal categories	110
5.2.2	Dimension larger than 2	111
5.2.3	Dimension 3 – spherical categories	114
5.2.4	Dimension 4 – ribbon categories	115
5.3	Wilson networks	117
5.3.1	Symmetric case	117
5.3.2	Non-symmetric case	118
5.4	Gauge fixing	121
5.4.1	Symmetric case	121
5.4.2	Non-symmetric case	122
6.	Topological Quantum Field Theory	125
6.1	Boundaries and gluing	125
6.1.1	Boundary circuit diagram	125
6.1.2	Gluing manifolds and diagrams	128
6.1.3	Sum over labelings	131
6.1.4	Wilson networks	132
6.2	Topological quantum field theory	133
6.2.1	The delta move	134
6.2.2	The anomaly move	136
6.2.3	Invariants of manifolds	138
6.2.4	Invariants of ribbon knots	139
6.2.5	Cobordisms	140
7.	Related Constructions	145
7.1	Spin networks	145
7.2	Spin foams and nj -symbols	147
7.3	Turaev-Viro invariant and Crane-Yetter invariant	150
7.4	The trace formalism	151
7.5	Modular categories and chain mail	153
8.	Applications to Lattice Models and Quantum Gravity	155
8.1	Two theories with $SU(2)$	155
8.2	The dual of non-abelian lattice gauge theory	157
8.3	The chiral model	158
8.4	Spin networks in loop quantum gravity	162
8.5	Spin foam models of quantum gravity	164
8.6	The Barrett-Crane model	166

8.7	Three-dimensional quantum supergravity	169
8.8	Generating field theory	172
8.8.1	From Feynman diagrams to triangulations	172
8.8.2	Generating BF theory	177
8.8.3	Beyond BF theory	179
8.8.4	A version of lattice gauge theory	180
8.8.5	The Barrett-Crane model	182
8.9	Renormalization	183
8.9.1	Standard renormalization	184
8.9.2	General discretizations	185
8.9.3	Cellular decompositions and moves	188
8.9.4	Examples	189

<i>References</i>	193
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<i>Index</i>	199
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