



World Scientific Lecture Notes in Physics – Vol. 74

LATTICE GAUGE THEORIES

An Introduction

Third Edition

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