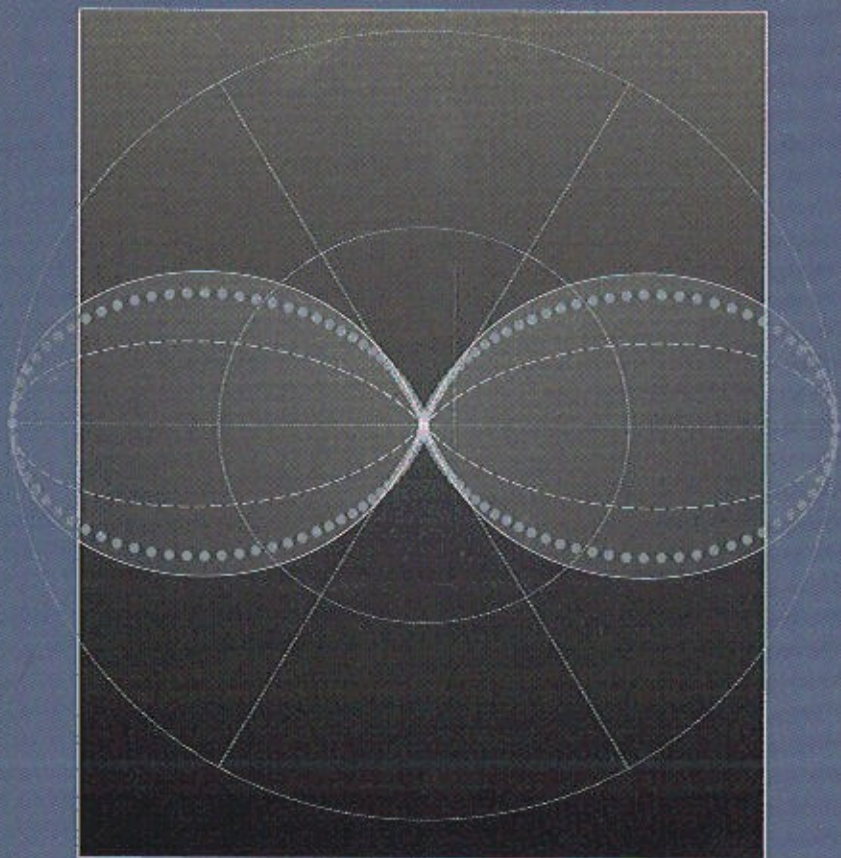


*C l a s s i c a l*  
**ELECTROMAGNETISM**



*J e r r o l d F r a n k l i n*

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