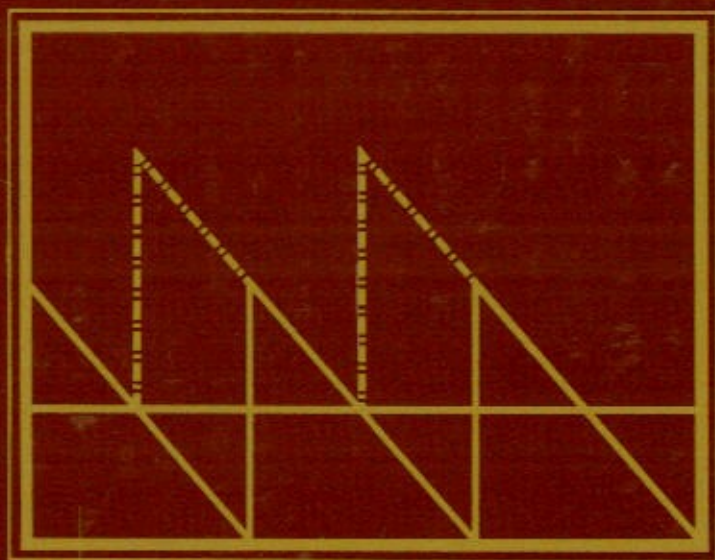


FOUNDATIONS OF INVENTORY MANAGEMENT



Paul H. Zipkin



McGRAW-HILL INTERNATIONAL EDITIONS

Management & Organization Series

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